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Barreto-Naehrig Curves draft-kasamatsu-bncurves-01

Abstract

Elliptic curves with pairings are useful tools for constructing cryptographic primitives. In this memo, we specify domain parameters of Barreto-Naehrig curves (BN-curves) [5]. The BN-curve is an elliptic curve suitable for pairings and allows us to achieve high security and efficiency of cryptographic schemes. This memo specifies domain parameters of two 254-bit BN-curves [1] [2] which allow us to obtain efficient implementations and domain parameters of 224, 256, 384, and 512-bit BN-curves which are compliant with ISO/IEC 15946-5[3]. Furthermore, this memo organizes differences between types of elliptic curves specified in ISO document and often used in open source software, which are called M-type and D-type, respectively[21].

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1. Introduction

Elliptic curves with a special map called a pairing or bilinear map allow cryptographic primitives to achieve functions or efficiency which cannot be realized by conventional mathematical tools. There are identity-based encryption (IBE), attribute-based encryption

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(ABE), ZSS signature, broadcast encryption (BE) as examples of such primitives. IBE realizes powerful management of public keys by allowing us to use a trusted identifier as a public key. ABE provides a rich decryption condition based on boolean functions and attributes corresponding to a secret key or a ciphertext. The ZSS signature gives a shorter size of signature than that of ECDSA. BE provides an efficient encryption procedure in a broadcast setting.

Some of these cryptographic schemes based on elliptic curves with pairings were proposed in the IETF (e.g. [6], [7], and [8]) and used in some protocols (e.g. [9], [10], [11], [12], and [13]). These cryptographic primitives will be used actively more in the IETF due to their functions or efficiency.

We need to choose an appropriate type of elliptic curve and parameters for the pairing-based cryptographic schemes, because the choice has great impact on security and efficiency of these schemes. However, an RFC on elliptic curves with pairings has not yet been provided in the IETF.

In this memo, we specify domain parameters of Barreto-Naehrig curve (BN-curve) [5]. The BN-curve allows us to achieve high security and efficiency with pairings due to an optimum embedding degree. This memo specifies domain parameters of two 254-bit BN-curves ([1] and [2]) because of these efficiencies. These BN-curves are known as efficient curves in academia and particularly provide efficient pairing computation which is generally slowest operation in pairingbased cryptography. There are optimized source codes of ([1] and [2]) as open source software ([19] and [20]), respectively. Furthermore, this memo specifies domain parameters of 224, 256, 384, and 512-bit curves which are compliant with ISO document [3] and organizes differences between types of elliptic curves specified in ISO document and used in open source software, which are called M-type and D-type respectively [21].

2. Requirements Terminology

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this memo are to be interpreted as described in [4].

3. Preliminaries

In this section, we introduce the definition of elliptic curve and bilinear map, notation used in this memo.

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3.1. Elliptic Curve

Throughout this memo, let p > 3 be a prime and F_p be a finite field. The curve defined by the following equation E is called an elliptic curve.

```
E : y^2 = x^3 + A^*x + B such that A, B are in F_p,
4 * A^3 + 27 * B^2 != 0 mod p
```

Solutions (x,y) for an elliptic curve E, as well as the point at infinity, are called F_p-rational points. The additive group is constructed by a well-defined operation in the set of F_p-rational points. Typically, the cyclic additive group with prime order q and base point G in $E(F_p)$ is used for the cryptographic applications. Furthermore, we define terminology used in this memo as follows.

O_E: the point at infinity over elliptic curve E.

#E(F_p): number of points on an elliptic curve E over F_p.

cofactor h: $h = \#E(F_p)/q$.

embedding degree k: minimum integer k such that r is a divisor of $q^k - 1$ and r^2 is not a divisor of $q^k - 1$

3.2. Bilinear Map

Let G_1 be an additive group of prime order p and let G_2 and G_T be additive and multiplicative groups, respectively, of the same order. Let P, Q be generators of G_1, G_2 respectively. We say that (G_1, G_2, G_T) are asymmetric bilinear map groups if there exists a bilinear map e: (G_1, G_2) -> G_T satisfying the following properties:

- 1. Bilinearity: for any S in G_1, for any T in G_2, for any a, b in Z_q , we have the relation $e(aS, bT) = e(S,T)^{ab}$.
- 2. Non-degeneracy: for any S in G_1, e(S,T) = 1 for any T in G_2
 only if S = O_E.
- 3. Computability: for any S in G_1, for any T in G_2, the bilinear map is efficiently computable.
- 4. There exists an efficient, publicly computable isomorphism I: G_2 -> G_1 such that I(Q) = P.

For BN-curves, G_1 is a q-order cyclic subgroup of $E(F_p)$ and G_2 is a subgroup of $E(F_{p^k})$, where k is the embedding degree of the

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curve. The group G_T is the set of q-th roots of unity in the finite field F_{p^k} .

4. Domain Parameter Specification

In this section, this memo specifies the domain parameters for two 254-bit elliptic curves which allow us to efficiently compute the operation of a pairing at high levels of security and domain parameters for 224, 256, 384, and 512-bit elliptic curves which are compliant with the ISO document [3].

4.1. Notation for Domain Parameters and Types of Sextic Twists

Here, we define notations for specifying domain parameters and explain types of pairing friendly curves.

Domain parameters of the elliptic curve $E(F_p)$ and $E(F_{p^{12}})$ are needed for computation of the pairing. Barreto and Naehrig proposed a method of point and pairing compression by using output of a map I from a sextic twist $E'(F_{p^{2}})$ to $E(F_{p^{12}})$ instead of $E(F_{p^{12}})$. Generally, this method is used with BN-curves. Hence, this memo follows the method. For the details of the method, refer to [5].

The pairing friendly curves are classified in two types, which are called D-type and M-type respectively [21]. The D-type sextic twist curve is defined by equation $y'^2 = x'^3 + b/s$ when elliptic curve $E(F_p)$ is set to be $y^2 = x^3 + b$ and represent of F_{p^12} is set to be $F_{p^2}[u]/(u^6 - s)$, where s is in $F_{p^2}^*$. Let z be a root of $u^6 - s$, where z is in F_{p^12} . The corresponding map I: $E'(F_{p^2}) = E(F_{p^12})$ is $(x', y') \to (z^2 * x', z^3 * y')$.

The M-type sextic twist curve is defined by equation $y'^2 = x'^3 + b$ * s when elliptic curve $E(F_p)$ is set to be $y^2 = x^3 + b$ and represent of F_{p^12} is set to be $F_{p^2}[u]/(u^6 - s)$, where s is in $F_{p^2}^*$. The corresponding map I: $E'(F_{p^2}) \rightarrow E(F_{p^12})$ is $(x', y') \rightarrow (x' * s^{-1}) * z^4, y' * s^{-1} * z^3)$, with $z^6 = s$.

These domain parameters are described in the following way.

Curve-ID is an identifier with which the curve can be referenced.

p_b is a prime specifying base field.

p_e is an irreducible polynomial specifying extension field.

For elliptic curve E

A and B are the coefficients of the equation $y^2 = x^3 + A * x + B$ mod p defining E.

G = (x,y) is the base point, i.e., a point with x and y being its x- and y-coordinates in E, respectively.

q is the prime order of the group generated by G.

h is the cofactor of G in E

For twisted curve E'

A' and B' are the coefficients of the equation $y^2 = x^3 + A' * x + B' \mod p$ defining E'.

G' = (x',y') is the base point, i.e., a point with x' and y' being its x'- and y'-coordinates in E', respectively.

q' is the prime order of the group generated by G'.

h' is the cofactor of G' in E'

4.2. Efficient Domain Parameters for 254-Bit-Curves

In this section, this memo specifies the domain parameters for two 254-bit elliptic curves which are more efficient than parameters of ISO document with D-type.

4.2.1. Domain Parameters by Beuchat et al.

Here, we describe the domain parameters for 254-bit elliptic curve[1] with D-type.

The domain parameters described in this subsection are defined by elliptic curve $E(F_p)$: $y^2 = x^3 + 5$ and sextic twist $E'(F_{p^2})$: $x'^3 + 5/s = x'^3 - u$, where $F_{p^2} = F_{p^2}[u]/(u^2 + 5)$, $F_{p^6} = F_{p^2}[v]/(v^3 - u)$, $F_{p^12} = F_{p^6}[w]/(w^2 - v)$, s = -5/u. We describe domain parameters of elliptic curves E and E'. For the details of these parameters, refer to [1].

Curve-ID: Fp254BNa

p_b = 0x2370fb049d410fbe4e761a9886e502417d023f40180000017e80600000
000001

A = 0

B = 5

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```
x = 1
     y = 0xd45589b158faaf6ab0e4ad38d998e9982e7ff63964ee1460342a592677cc
     cb0
     q = 0x2370fb049d410fbe4e761a9886e502411dc1af70120000017e8060000000
      0001
     h = 1
     Curve-ID: Fp254n2BNa
     p_b = 0x2370fb049d410fbe4e761a9886e502417d023f40180000017e80600000
     000001
     p_e = u^2 + 5 \text{ over } p_b
     A' = 0
     B' = - u
     x' = 0x19b0bea4afe4c330da93cc3533da38a9f430b471c6f8a536e81962ed967
      909b5 + (0xalcf585585a61c6e9880b1f2a5c539f7d906fff238fa6341e1de1a2
     e45c3f72) u
     y' = 0x17abd366ebbd65333e49c711a80a0cf6d24adf1b9b3990eedcc91731384
     d2627 +
      (0x0ee97d6de9902a27d00e952232a78700863bc9aa9be960C32f5bf9fd
      0a32d345) u
     q' = 0x2370fb049d410fbe4e761a9886e502411dc1af70120000017e806000000
      00001
     h' = 0x2370fb049d410fbe4e761a9886e50241dc42cf101e0000017e806000000
     00001
4.2.2. Domain Parameters by Aranha et al.
  Here, we describe the domain parameters for 254-bit elliptic curve
  [2] with D-type.
  The domain parameters described in this subsection are defined by
  elliptic curve E(F_p) : y^2 = x^3 + 2 and sextic twist E'(F_{p^2}) :
  x'^3 + 2/s = x'^3 + 1 - u, where F_{p^2} = F_p [u]/(u^2 + 1),
```

curves E and E'. For the details of these parameters, refer to [2].

 $F_{p^6} = F_{p^2} [v]/(v^3 - (1+u)), F_{p^12} = F_{p^6} [w]/(w^2 - v), 1/s = 1/(1 + u).$ We describes domain parameters of elliptic

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Curve-ID: Fp254BNb

p_b = 0x2523648240000001ba344d80000000861210000000013a700000000 000013

A = 0

B = 2

x = 0x2523648240000001ba344d80000000861210000000013a7000000000 0012

y = 1

q = 0x2523648240000001ba344d800000007ff9f80000000010a1000000000 000d

h = 1

Curve-ID: Fp254n2BNb

p_b = 0x2523648240000001ba344d80000000861210000000013a700000000 000013

 $p_e = u^2 + 1$ over p_b

A' = 0

B' = 1 + (0x2523648240000001ba344d800000008612100000000013a70000 000000012) u

x' = 0x061a10bb519eb62feb8d8c7e8c61edb6a4648bbb4898bf0d91ee4224c80
3fb2b +(0x0516aaf9ba737833310aa78c5982aa5b1f4d746bae3784b70d8c34c1
e7d54cf3)u

y' = 0x021897a06baf93439a90e096698c822329bd0ae6bdbe09bd19f0e07891cd2b9a + (0x0ebb2b0e7c8b15268f6d4456f5f38d37b09006ffd739c9578a2d1ae c6b3ace9b) u

h' = 0x2523648240000001ba344d800000008c2a280000000016ad00000000 00019

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4.3. Domain Parameters Based on ISO Document

Here, we describe the domain parameters for 224, 256, 384, and 512-bit elliptic curves which are compliant with the ISO document and are based on M-type. The domain parameters described in below subsections are defined by Elliptic curve $E(F_p)$: $y^2 = x^3 + 3$ and sextic twist $E'(F_{p^2})$: $y'^2 = x'^3 + 3 * s$, where $F_{p^2} = F_p[X] / (X^2 + 1)$, $F_{p^12} = F_{p^2}[X]/(X^6 - s)$, s = 1 + X. We describe domain parameters of elliptic curves E. Detailed information on these domain parameters is given in [3].

4.3.1. Domain Parameters for 224-Bit Curves

Curve-ID: Fp224BN

- A = 0
- B = 3
- x = 1
- y = 2
- h = 1

4.3.2. Domain Parameters for 256-Bit Curves

Curve-ID: Fp256BN

- A = 0
- B = 3
- x = 1
- y = 2

h = 1

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```
4.3.3. Domain Parameters for 384-Bit Curves
```

Curve-ID: Fp384BN

- A = 0
- B = 3
- x = 1
- y = 2

h = 1

4.3.4. Domain Parameters for 512-Bit Curves

Curve-ID: Fp512BN

- A = 0
- B = 3
- x = 1
- y = 2

h = 1

4.3.5. Differences between D-Type and M-Type on ISO parameters

Although ISO document is based on M-type, open source software are often based on D-type. We need to be aware of the differences. Hence we also describe elliptic curves with D-type based on ISO document [3]. The elliptic curve $E(F_p)$ is defined by equation $y^2 =$

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x^3 + 3 and the sextic twist $E'(F_{p^2})$ is defined by $y'^2 = x'^3 + 3/s$, where $F_{p^2} = F_p[X]/(X^2 + 1)$, $F_{p^12} = F_{p^2}[X]/(X^6 - s)$, 1/s = -8 + 8 * i, $i = X^2 + 1$. Detailed information on these domain parameters is given in [5].

5. Object Identifiers

We need to define the following object identifiers. Which organization is suitable for the allotment of these object identifiers?

Fp254BNa OBJECT IDENTIFIER ::= {TBD}
Fp254n2BNa OBJECT IDENTIFIER ::= {TBD}
Fp254n2BNb OBJECT IDENTIFIER ::= {TBD}
Fp254n2BNb OBJECT IDENTIFIER ::= {TBD}
Fp224BN OBJECT IDENTIFIER ::= {TBD}
Fp256BN OBJECT IDENTIFIER ::= {TBD}
Fp384BN OBJECT IDENTIFIER ::= {TBD}
Fp512BN OBJECT IDENTIFIER ::= {TBD}

6. Security Considerations

Elliptic curves which are specified in this memo have hardness of the problems below and enough security margin against the attacks below.

Pairing-based cryptographic primitives are often based on the hardness of the following problems, so when the elliptic curves from this document are used in such schemes, these problems would apply. (For details of problems, refer to section 2 of [14].)

The elliptic curve discrete logarithm problem (ECDLP)

The elliptic curve computational Diffie-Hellman problem (ECDHP)

The bilinear Diffie-Hellman problem (BDHP)

The elliptic curve discrete logarithm problem with auxiliary inputs (ECDLP with auxiliary inputs)

Algorithms to efficiently solve the problems above, aside from special cases, are unknown. Mainly, there are Pollard-rho algorithm

[16] against point of an elliptic curve and Number Field Sieve method [15] against output of pairing as generic attacks against elliptic curve with pairing.

The Smart, Semaev, and Sato-Araki algorithm [17], and Cheon algorithm [14] are main algorithms which improve efficiency in specific cases. The Smart-Semaev algorithm and Sato-Araki algorithm are polynotmial time algorithms against the ECDLP in the case where $\#E(F_{p})$ equals to p. These algorithms are independently proposed. Cheon algorithm [14] is against the ECDLP with auxiliary inputs. It is prevented by satisfy the following condition, where n is the order of the curve.

there is no divisor d of n - 1 s.t. $(\log n)^2 < d < n^{1/2}$ and there is no divisor e of n + 1 s.t. $(\log n)^2 < e < n^{1/2}$

Table 1 shows the security level of elliptic curves described in this memo ([1], [2]). Schemes based on the elliptic curves must be combined with cryptographic primitives which have similar or greater security level than the scheme.

Curve-ID	Security Level (bits)		
Fp224BN	112		
Fp254BNa	128		
Fp254BNb	128		
Fp256BN	128		
Fp384BN	128		
Fp512BN	128		

Table 1: security level of elliptic curve specified in this memo

7. Acknowledgements

This memo was inspired by the content and structure of [18].

8. Change log

NOTE TO RFC EDITOR: Please remove this section in before final RFC publication.

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