                     XMSS: Extended Hash-Based Signatures
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Abstract

   This note describes the eXtended Merkle Signature Scheme (XMSS), a
   hash-based digital signature system. It follows existing
   descriptions in scientific literature. The note specifies the WOTS+
   one-time signature scheme, a single-tree (XMSS) and a multi-tree
   variant (XMSS^MT) of XMSS. Both variants use WOTS+ as a main
   building block. XMSS provides cryptographic digital signatures
   without relying on the conjectured hardness of mathematical problems.
   Instead, it is proven that it only relies on the properties of
   cryptographic hash functions. XMSS provides strong security
   guarantees and is even secure when the collision resistance of the
   underlying hash function is broken. It is suitable for compact
   implementations, relatively simple to implement, and naturally
   resists side-channel attacks. Unlike most other signature systems,
   hash-based signatures withstand attacks using quantum computers.

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1. Introduction

   A (cryptographic) digital signature scheme provides asymmetric
   message authentication. The key generation algorithm produces a key
   pair consisting of a private and a public key. A message is signed
   using a private key to produce a signature. A message/signature pair
   can be verified using a public key. A One-Time Signature (OTS)
   scheme allows using a key pair to sign exactly one message securely.
   A many-time signature system can be used to sign multiple messages.

   One-Time Signature schemes, and Many-Time Signature (MTS) schemes
   composed of them, were proposed by Merkle in 1979 [Merkle79]. They
   were well-studied in the 1990s and have regained interest from 2006
   onwards because of their resistance against quantum-computer-aided
   attacks. These kinds of signature schemes are called hash-based
   signature schemes as they are built out of a cryptographic hash

function. Hash-based signature schemes generally feature small
private and public keys as well as fast signature generation and
verification but large signatures and relatively slow key generation.
In addition, they are suitable for compact implementations that
benefit various applications and are naturally resistant to most
kinds of side-channel attacks.

Some progress has already been made toward standardizing and
introducing hash-based signatures. McGrew and Curcio have published
an Internet-Draft [DC16] specifying the Lamport-Diffie-Winternitz-
Merkle (LDWM) scheme, also taking into account subsequent adaptations
by Leighton and Micali. Independently, Buchmann, Dahmen and Huelsing
have proposed XMSS [BDH11], the eXtended Merkle Signature Scheme,
offering better efficiency and a modern security proof. Very
recently, the stateless hash-based signature scheme SPHINCS was
introduced [BHH15], with the intent of being easier to deploy in
current applications. A reasonable next step toward introducing
hash-based signatures would be to complete the specifications of the
basic algorithms - LDWM, XMSS, SPHINCS and/or variants [Kaliski15].

The eXtended Merkle Signature Scheme (XMSS) [BDH11] is the latest
stateful hash-based signature scheme. It has the smallest signatures
out of such schemes and comes with a multi-tree variant that solves
the problem of slow key generation. Moreover, it can be shown that
XMSS is secure, making only mild assumptions on the underlying hash
function. Especially, it is not required that the cryptographic hash
function is collision-resistant for the security of XMSS.

This document describes a single-tree and a multi-tree variant of
XMSS. It also describes WOTS+, a variant of the Winternitz OTS
scheme introduced in [Huelsing13] that is used by XMSS. The schemes
are described with enough specificity to ensure interoperability
between implementations.

This document is structured as follows. Notation is introduced in
Section 2. Section 3 describes the WOTS+ signature system. MTS
schemes are defined in Section 4: the eXtended Merkle Signature
Scheme (XMSS) in Section 4.1, and its Multi-Tree variant (XMSS^MT) in
Section 4.2. Parameter sets are described in Section 5. Section 6
describes the rationale behind choices in this note. The IANA
registry for these signature systems is described in Section 7.
Finally, security considerations are presented in Section 8.

1.1. Conventions Used In This Document

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT",
"SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this
document are to be interpreted as described in [RFC2119].

2. Notation

2.1. Data Types

Bytes and byte strings are the fundamental data types. A byte is a sequence of eight bits. A single byte is denoted as a pair of hexadecimal digits with a leading "0x". A byte string is an ordered sequence of zero or more bytes and is denoted as an ordered sequence of hexadecimal characters with a leading "0x". For example, 0xe534f0 is a byte string of length 3. An array of byte strings is an ordered, indexed set starting with index 0 in which all byte strings have identical length. We assume big-endian representation for any data types or structures.

2.2. Operators

When a and b are integers, mathematical operators are defined as follows:

^ : a ^ b denotes the result of a raised to the power of b.

* : a * b denotes the product of a and b. This operator is sometimes used implicitly in the absence of ambiguity, as in usual mathematical notation.

/ : a / b denotes the quotient of a by b.

% : a % b denotes the non-negative remainder of the integer division of a by b.

+ : a + b denotes the sum of a and b.

- : a - b denotes the difference of a and b.

The standard order of operations is used when evaluating arithmetic expressions.

Arrays are used in the common way, where the ith element of an array A is denoted A[i]. Byte strings are treated as arrays of bytes where necessary: If X is a byte string, then X[i] denotes its ith byte, where X[0] is the leftmost byte.

If A and B are byte strings of equal length, then:

A AND B denotes the bitwise logical conjunction operation.

A XOR B denotes the bitwise logical exclusive disjunction operation.

When B is a byte and i is an integer, then B >> i denotes the logical right-shift operation. Similarly, B << i denotes the logical left-shift operation.

If X is an x-byte string and Y a y-byte string, then X || Y denotes the concatenation of X and Y, with X || Y = X[0] ... X[x-1] Y[0] ... Y[y-1].

2.3. Functions

If x is a non-negative real number, then we define the following functions:

   ceil(x) : returns the smallest integer greater than or equal to x.

   floor(x) : returns the largest integer less than or equal to x.

   lg(x) : returns the logarithm to base 2 of x.

2.4. Integer to Byte Conversion

If x and y are non-negative integers, we define Z = toByte(x, y) to be the y-byte string containing the binary representation of x in big-endian byte-order.

2.5. Hash Function Address Scheme

The schemes described in this document randomize each hash function call. This means that aside of the initial message digest, for each hash function call a different key and different bitmask is used. These values are pseudorandomly generated using a pseudorandom generator that takes a seed S and a 32-byte address A. The latter is used to select the A-th n-byte block from the PRG output where n is the security parameter. Here we explain the structure of address A. We explain the construction of the addresses in the following sections where they are used.

The schemes in the next two sections use two kinds of hash functions parameterized by security parameter n. For the hash tree constructions a hash function that maps 2n-byte inputs and an n-byte key to n-byte outputs is used. To randomize this function, 3n bytes are needed - n bytes for the key and 2n bytes for a bitmask. For the one-time signature scheme constructions a hash function that maps n-byte inputs and n-byte keys to n-byte outputs is used. To randomize this function, 2n bytes are needed - n bytes for the key and n bytes for a bitmask. Consequently, three addresses are needed for the first function and two addresses for the second one.

There are three different address formats for the different use cases. One format for the hashes used in one-time signature schemes, one for hashes used within the main Merkle-tree construction, and one for hashes used in the L-trees. The latter being used to compress one-time public keys. All these formats share as much format as possible. In the following we describe these formats in detail.

The structure of an address complies with byte borders, as well as with word borders, with a word being 32 bits long in this context. Only the tree address is too long to fit a single word but matches a double word. An address is structured as follows. It always starts with a layer address of 32 bits in the most significant bits, followed by a tree address of 64 bits. Both addresses are needed for the multi-tree variant (see Section 4.2) and describe the position of a tree within a multi-tree. They are therefore set to zero in case of single-tree applications. For multi-tree hash-based signatures the layer address describes the height of a tree within the multi-tree starting from height zero for trees at the bottom layer. The tree address describes the position of a tree within a layer of a multi-tree starting with index zero for the leftmost tree. Next, following a zero padding of seven bits, the next bit specifies whether it is an OTS or a hash tree address. This OTS bit is set to zero for a hash tree and to one for an OTS hash address.

We first describe the OTS address case as the hash tree case again splits into two cases. In this case, the OTS bit is followed by a zero padding of 24 bits. The padding is followed by a 32-bit OTS address that encodes the index of the OTS key pair within the tree. The next 32 bits encode the chain address followed by 32 bits that encode the address of the hash function call within the chain. The next 31 bits contain a zero padding. The last bit is the key bit, used to generate two different addresses for one hash function call. The bit is set to one to generate the key. To generate the n-byte bitmask, the key bit is set to zero.

```
          An OTS hash address
+------------------------+
| layer address  (32 bit)|
+------------------------+
| tree address   (64 bit)|
+------------------------+
| Padding = 0     (7 bit)|
+------------------------+
| OTS bit = 1     (1 bit)|
+------------------------+
| Padding = 0    (24 bit)|
+------------------------+
| OTS address    (32 bit)|
+------------------------+
| chain address  (32 bit)|
+------------------------+
| hash address   (32 bit)|
+------------------------+
| Padding = 0    (31 bit)|
+------------------------+
| key bit         (1 bit)|
+------------------------+
```

Now we describe the hash tree address case. This case again splits
into two. The OTS bit is followed by a zero padding of 23 bits and
an L-tree bit. This bit is set to one in case of an L-tree and set
to zero for main tree nodes. We now discuss the L-tree case, which
means that the L-tree bit is set to one. In that case the L-tree bit
is followed by an L-tree address of 32 bits that encodes the index of
the leaf computed with this L-tree. The next 32 bits encode the
height of the node inside the L-tree and the following 32 bits encode
the index of the node at that height, inside the L-tree. After a
zero padding of 30 bits, the two last bits are used to generate three
different addresses for one node. The first of these bits (the key
bit) is set to one to generate the key. In that case the next bit
(the block bit) is always zero. To generate the 2n-byte bitmask, the
key bit is set to zero. The most significant n bytes are generated
using the address with the block bit set to zero. The least
significant bytes are generated using the address with the block bit
set to one.

```
                   An L-tree address
              +-----------------------+
              | layer address  (32 bit)|
              +-----------------------+
              | tree address   (64 bit)|
              +-----------------------+
              | Padding = 0     (7 bit)|
              +-----------------------+
              | OTS bit = 0     (1 bit)|
              +-----------------------+
              | Padding = 0    (23 bit)|
              +-----------------------+
              | L-tree bit = 1  (1 bit)|
              +-----------------------+
              | L-tree address (32 bit)|
              +-----------------------+
              | tree height    (32 bit)|
              +-----------------------+
              | tree index     (32 bit)|
              +-----------------------+
              | Padding = 0    (30 bit)|
              +-----------------------+
              | key bit         (1 bit)|
              +-----------------------+
              | block bit       (1 bit)|
              +-----------------------+
```

We now describe the remaining format for the main tree hash
addresses. In this case the L-tree bit is set to zero, followed by a
zero padding of 32 bits. The next 32 bits encode the height of the
tree node to be computed within the tree, followed by 32 bits that
encode the index of this node at that height. After a zero padding
of 30 bits, the two last bits are used to generate three different
addresses for one node as described for the L-tree case. The first
of these bits is set to one to generate the key. In that case the
latter bit is always zero. To generate the 2n-byte bitmask, the key
bit is set to zero. The most significant n bytes are generated using
the address with the block bit set to zero. The least significant
bytes are generated using the address with the block bit set to one.

```
             A hash tree address
         +------------------------+
         | layer address  (32 bit)|
         +------------------------+
         | tree address   (64 bit)|
         +------------------------+
         | Padding = 0     (7 bit)|
         +------------------------+
         | OTS bit = 0     (1 bit)|
         +------------------------+
         | Padding = 0    (23 bit)|
         +------------------------+
         | L-tree bit = 0  (1 bit)|
         +------------------------+
         | Padding = 0    (32 bit)|
         +------------------------+
         | tree height    (32 bit)|
         +------------------------+
         | tree index     (32 bit)|
         +------------------------+
         | Padding = 0    (30 bit)|
         +------------------------+
         | key bit         (1 bit)|
         +------------------------+
         | block bit       (1 bit)|
         +------------------------+
```
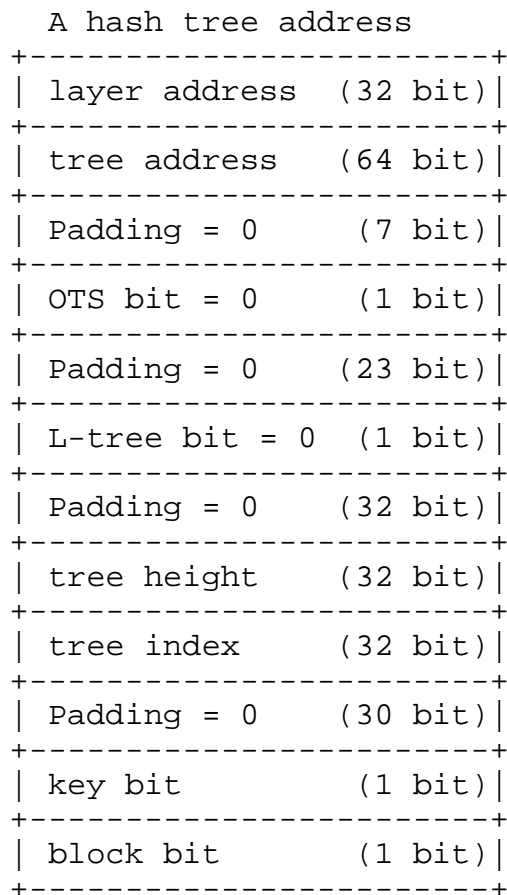
All fields within these addresses encode unsigned integers. When describing the generation of addresses we use setter-methods that take positive integers and set the bits of a field to the binary representation of that integer of the length of the field. We also assume that setting the L-tree bit to zero, does also set the other padding block to zero.

2.6. Strings of Base w Numbers

A byte string can be considered as a string of base w numbers, i.e. integers in the set {0, ... , w - 1}. The correspondence is defined by the function base_w(X, w, out_len) as follows. If X is a len_X-byte string, and w is a member of the set {4, 16}, then base_w(X, w, out_len) outputs an array of out_len integers between 0 and w - 1. The length out_len is REQUIRED to be less than or equal to 8 * len_X / lg(w).

Algorithm 1: base_w

```
Input: len_X-byte string X, int w, output length out_len
Output: out_len int array basew

    int in = 0;
    int out = 0;
    unsigned int total = 0;
    int bits = 0;
    int consumed;

    for ( consumed = 0; consumed < out_len; consumed++ ) {
        if ( bits == 0 ) {
            total = X[in];
            in++;
            bits += 8;
        }
        bits -= lg(w);
        basew[out] = (total >> bits) AND (w - 1);
        out++;
    }
    return basew;
```

For example, if X is the (big-endian) byte string 0x1234, then
base_w(X, 16, 4) returns the array a = {1, 2, 3, 4}.

```
                X (represented as bits)
     +--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+
     | 0| 0| 0| 1| 0| 0| 1| 0| 0| 0| 1| 1| 0| 1| 0| 0|
     +--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+
           X[0]              |           X[1]

           X (represented as base 16 numbers)
     +----------+----------+----------+----------+
     |    1     |    2     |    3     |    4     |
     +----------+----------+----------+----------+

                 base_w(X, 16, 4)
     +----------+----------+----------+----------+
     |    1     |    2     |    3     |    4     |
     +----------+----------+----------+----------+
        a[0]       a[1]       a[2]       a[3]
```

2.7. Member Functions

To simplify algorithm descriptions, we assume the existence of member
functions. If a complex data structure like a public key PK contains
a value X then getX(PK) returns the value of X for this public key.

Accordingly, setX(PK, X, Y) sets value X in PK to the value hold by Y. Since camelCase is used for member function names, a value z may be referred to as Z in the function name, e.g. getZ.

3. Primitives

3.1. WOTS+ One-Time Signatures

This section describes the WOTS+ one-time signature system, in a version similar to [Huelsing13]. WOTS+ is a one-time signature scheme; while a private key can be used to sign any message, each private key MUST be used only once to sign a single message. In particular, if a secret key is used to sign two different messages, the scheme becomes insecure.

The section starts with an explanation of parameters. Afterwards, the so-called chaining function, which forms the main building block of the WOTS+ scheme, is explained. A description of the algorithms for key generation, signing and verification follows. Finally, pseudorandom key generation is discussed.

3.1.1. WOTS+ Parameters

WOTS+ uses the parameters n, and w; they all take positive integer values. These parameters are summarized as follows:

   n : the message length as well as the length of a secret key, public key, or signature element in bytes.

   w : the Winternitz parameter; it is a member of the set {4, 16}.

The parameters are used to compute values len, len_1 and len_2:

   len : the number of n-byte string elements in a WOTS+ secret key, public key, and signature. It is computed as len = len_1 + len_2, with len_1 = ceil(8n / lg(w)) and len_2 = floor(lg(len_1 * (w - 1)) / lg(w)) + 1.

The value of n is determined by the cryptographic hash function used for WOTS+. The hash function is chosen to ensure an appropriate level of security. The value of n is the input length that can be processed by the signing algorithm. It is often the length of a message digest. The parameter w can be chosen from the set {4, 16}. A larger value of w results in shorter signatures but slower overall signing operations; it has little effect on security. Choices of w are limited to the values 4 and 16 since these values yield optimal trade-offs and easy implementation.

WOTS+ parameters are implicitly included in algorithm inputs as needed.

3.1.1.1. WOTS+ Functions

The WOTS+ algorithm uses a keyed cryptographic hash function F. F accepts and returns byte strings of length n using keys of length n. Security requirements on F are discussed in Section 8. In addition, WOTS+ uses a pseudorandom function PRF. PRF takes as input an n-byte key and a 32-byte index and generates pseudorandom outputs of length n. Security requirements on PRF are discussed in Section 8.

3.1.2. WOTS+ Chaining Function

The chaining function (Algorithm 2) computes an iteration of F on an n-byte input using outputs of PRF. It takes an OTS hash address as input. This address will have the first six 32-bit words set to encode the address of this chain. In each iteration, PRF is used to generate a key for F and a bitmask that is XORed to the intermediate result before it is processed by F. In the following, ADRS is a 32-byte OTS hash address as specified in Section 2.5 and SEED is an n-byte string. To generate the keys and bitmasks, PRF is called with SEED as key and ADRS as input. The chaining function takes as input an n-byte string X, a start index i, a number of steps s, as well as ADRS and SEED. The chaining function returns as output the value obtained by iterating F for s times on input X, using the outputs of PRF.

Algorithm 2: chain - Chaining Function

```
  Input: Input string X, start index i, number of steps s, address
  ADRS, seed SEED
  Output: value of F iterated s times on X

  if ( s == 0 ) {
    return X;
  }
  if ( (i + s) > w - 1 ) {
    return NULL;
  }
  byte[n] tmp = chain(X, i, s - 1, SEED, ADRS);
  ADRS.setHashAddress(i + s - 1);
  ADRS.setKeyBit(0);
  BM = PRF(SEED, ADRS);
  ADRS.setKeyBit(1);
  KEY = PRF(SEED, ADRS);
  tmp = F(KEY, tmp XOR BM);
  return tmp;
```

3.1.3. WOTS+ Private Key

   The private key in WOTS+, denoted by sk, is a length len array of
   n-byte strings. This private key MUST be only used to sign at most
   one message. Each n-byte string MUST either be selected randomly
   from the uniform distribution or using a cryptographically secure
   pseudorandom procedure. In the latter case, the security of the used
   procedure MUST at least match that of the WOTS+ parameters used. For
   a further discussion on pseudorandom key generation see the end of
   this section. The following pseudocode (Algorithm 3) describes an
   algorithm for generating sk.

   Algorithm 3: WOTS_genSK - Generating a WOTS+ Private Key

      Input: /
      Output: WOTS+ secret key sk

      for (i = 0; i < len; i++) {
        initialize sk[i] with a uniformly random n-byte string;
      }
      return sk;

3.1.4. WOTS+ Public Key

   A WOTS+ key pair defines a virtual structure that consists of len
   hash chains of length w. The len n-byte strings in the secret key
   each define the start node for one hash chain. The public key
   consists of the end nodes of these hash chains. Therefore, like the
   secret key, the public key is also a length len array of n-byte
   strings. To compute the hash chain, the chaining function (Algorithm
   2) is used. An OTS hash address ADRS and a seed SEED have to be
   provided by the calling algorithm. This address will encode the
   address of the WOTS+ key pair within a greater structure. Hence, a
   WOTS+ algorithm MUST NOT manipulate any other parts of ADRS than the
   last three 32-bit words. Please note that the SEED used here is
   public information also available to a verifier. The following
   pseudocode (Algorithm 4) describes an algorithm for generating the
   public key pk, where sk is the private key.

Algorithm 4: WOTS_genPK - Generating a WOTS+ Public Key From a
Private Key

```
Input: WOTS+ secret key sk, address ADRS, seed SEED
Output: WOTS+ public key pk

for ( i = 0; i < len; i++ ) {
  ADRS.setChainAddress(i);
  pk[i] = chain(sk[i], 0, w - 1, SEED, ADRS);
}
return pk;
```

3.1.5. WOTS+ Signature Generation

A WOTS+ signature is a length len array of n-byte strings. The WOTS+
signature is generated by mapping a message to len integers between 0
and w - 1. To this end, the message is transformed into len_1 base w
numbers using the base_w function defined in Section 2.6. Next, a
checksum is computed and appended to the transformed message as len_2
base w numbers using the base_w function. Each of the base w
integers is used to select a node from a different hash chain. The
signature is formed by concatenating the selected nodes. An OTS hash
address ADRS and a seed SEED have to be provided by the calling
algorithm. This address will encode the address of the WOTS+ key
pair within a greater structure. Hence, a WOTS+ algorithm MUST NOT
manipulate any other parts of ADRS than the last three 32-bit words.
Please note that the SEED used here is public information also
available to a verifier. The pseudocode for signature generation is
shown below (Algorithm 5), where M is the message and sig is the
resulting signature.

   Algorithm 5: WOTS_sign - Generating a signature from a private key
   and a message

      Input: Message M, WOTS+ secret key sk, address ADRS, seed SEED
      Output: WOTS+ signature sig

      csum = 0;

      // convert message to base w
      msg = base_w(M, w, len_1);

      // compute checksum
      for (i = 0; i < len_1; i++) {
            csum = csum + w - 1 - msg[i];
      }

      // Convert csum to base w
      csum = csum << (8 - ((len_2 * lg(w)) % 8));
      len_2_bytes = ceil((len_2 * lg(w)) / 8);
      msg = msg || base_w(toByte(csum, len_2_bytes), w, len_2);
      for (i = 0; i < len; i++) {
            ADRS.setChainAddress(i);
            sig[i] = chain(sk[i], 0, msg[i], SEED, ADRS);
      }
      return sig;

   The data format for a signature is given below.

   WOTS+ Signature

```
         +-------------------------------+
         |                               |
         |          sig_ots[0]           |    n bytes
         |                               |
         +-------------------------------+
         |                               |
         ~             ....              ~
         |                               |
         +-------------------------------+
         |                               |
         |        sig_ots[len - 1]       |    n bytes
         |                               |
         +-------------------------------+
```
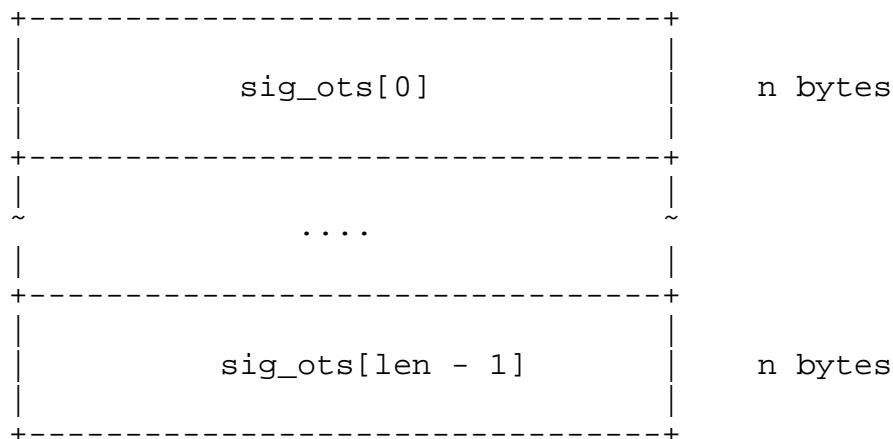
3.1.6. WOTS+ Signature Verification

   In order to verify a signature sig on a message M, the verifier
   computes a WOTS+ public key value from the signature. This can be
   done by "completing" the chain computations starting from the
   signature values, using the base w values of the message hash and its
   checksum. This step, called WOTS_pkFromSig, is described below in
   Algorithm 6. The result of WOTS_pkFromSig is then compared to the
   given public key. If the values are equal, the signature is
   accepted. Otherwise, the signature MUST be rejected. An OTS hash
   address ADRS and a seed SEED have to be provided by the calling
   algorithm. This address will encode the address of the WOTS+ key
   pair within a greater structure. Hence, a WOTS+ algorithm MUST NOT
   manipulate any other parts of ADRS than the last three 32-bit words.
   Please note that the SEED used here is public information also
   available to a verifier.

   Algorithm 6: WOTS_pkFromSig - Computing a WOTS+ public key from a
   message and its signature

      Input: Message M, WOTS+ signature sig, address ADRS, seed SEED
      Output: 'Temporary' WOTS+ public key tmp_pk

      csum = 0;

      // convert message to base w
      msg = base_w(M, w, len_1);

      // compute checksum
      for (i = 0; i < len_1; i++) {
           csum = csum + w - 1 - msg[i];
      }

      // Convert csum to base w
      csum = csum << (8 - ((len_2 * lg(w)) % 8));
      len_2_bytes = ceil((len_2 * lg(w)) / 8);
      msg = msg || base_w(toByte(csum, len_2_bytes), w, len_2);
      for (i = 0; i < len; i++) {
           ADRS.setChainAddress(i);
           tmp_pk[i] = chain(sig[i], msg[i], w - 1 - msg[i], SEED, ADRS);
      }
      return tmp_pk;

   Note: XMSS uses WOTS_pkFromSig to compute a public key value and
   delays the comparison to a later point.

3.1.7. Pseudorandom Key Generation

   An implementation MAY use a cryptographically secure pseudorandom
   method to generate the secret key from a single n-byte value. For
   example, the method suggested in [BDH11] and explained below MAY be
   used. Other methods MAY be used. The choice of a pseudorandom
   method does not affect interoperability, but the cryptographic
   strength MUST match that of the used WOTS+ parameters.

   The advantage of generating the secret key elements from a random
   n-byte string is that only this n-byte string needs to be stored
   instead of the full secret key. The key can be regenerated when
   needed. The suggested method from [BDH11] can be described using
   PRF. During key generation a uniformly random n-byte string S is
   sampled from a secure source of randomness. This string S is stored
   as secret key. The secret key elements are computed as sk[i] =
   PRF(S, toByte(i, 32)) whenever needed. Please note that this seed S
   MUST be different from the seed SEED used to randomize the hash
   function calls. Also, this seed S MUST be kept secret.

4. Schemes

   In this section, the eXtended Merkle Signature Scheme (XMSS) is
   described using WOTS+. XMSS comes in two flavors: First, a single-
   tree variant (XMSS) and second a multi-tree variant (XMSS^MT). Both
   allow combining a large number of WOTS+ key pairs under a single
   small public key. The main ingredient added is a binary hash tree
   construction. XMSS uses a single hash tree while XMSS^MT uses a tree
   of XMSS key pairs.

4.1. XMSS: eXtended Merkle Signature Scheme

   XMSS is a method for signing a potentially large but fixed number of
   messages. It is based on the Merkle signature scheme. XMSS uses
   four cryptographic components: WOTS+ as OTS method, two additional
   cryptographic hash functions H and H_msg, and a pseudorandom function
   PRF. One of the main advantages of XMSS with WOTS+ is that it does
   not rely on the collision resistance of the used hash functions but
   on weaker properties. Each XMSS public/private key pair is
   associated with a perfect binary tree, every node of which contains
   an n-byte value. Each tree leaf contains a special tree hash of a
   WOTS+ public key value. Each non-leaf tree node is computed by first
   concatenating the values of its child nodes, computing the XOR with a
   bitmask, and applying the keyed hash function H to the result. The
   bitmasks and the keys for the hash function H are generated from a
   (public) seed that is part of the public key using the pseudorandom
   function PRF. The value corresponding to the root of the XMSS tree
   forms the XMSS public key together with the seed.

To generate a key pair that can be used to sign 2^h messages, a tree of height h is used. XMSS is a stateful signature scheme, meaning that the secret key changes with every signature generation. To prevent one-time secret keys from being used twice, the WOTS+ key pairs are numbered from 0 to (2^h) - 1 according to the related leaf, starting from index 0 for the leftmost leaf. The secret key contains an index that is updated with every signature generation, such that it contains the index of the next unused WOTS+ key pair.

A signature consists of the index of the used WOTS+ key pair, the WOTS+ signature on the message and the so-called authentication path. The latter is a vector of tree nodes that allow a verifier to compute a value for the root of the tree starting from a WOTS+ signature. A verifier computes the root value and compares it to the respective value in the XMSS public key. If they match, the signature is valid. The XMSS secret key consists of all WOTS+ secret keys and the actual index. To reduce storage, a pseudorandom key generation procedure, as described in [BDH11], MAY be used. The security of the used method MUST at least match the security of the XMSS instance.

4.1.1. XMSS Parameters

XMSS has the following parameters:

   h : the height (number of levels - 1) of the tree

   n : the length in bytes of the message digest as well as of each node

   w : the Winternitz parameter as defined for WOTS+ in Section 3.1

There are 2^h leaves in the tree.

For XMSS and XMSS^MT, secret and public keys are denoted by SK and PK. For WOTS+, secret and public keys are denoted by sk and pk, respectively. XMSS and XMSS^MT signatures are denoted by Sig. WOTS+ signatures are denoted by sig.

XMSS and XMSS^MT parameters are implicitly included in algorithm inputs as needed.

4.1.2. XMSS Hash Functions

Besides the cryptographic hash function F and the pseudorandom function PRF required by WOTS+, XMSS uses two more functions:

   A cryptographic hash function H. H accepts n-byte keys and byte strings of length (2 * n) and returns an n-byte string.

A cryptographic hash function H_msg. H_msg accepts 3n-byte keys
and byte strings of arbitrary length and returns an n-byte string.

4.1.3. XMSS Private Key

An XMSS private key SK contains 2^h WOTS+ private keys, the leaf
index idx of the next WOTS+ private key that has not yet been used,
SK_PRF, an n-byte key to generate pseudorandom values for randomized
message hashing, the n-byte value root, which is the root node of the
tree and SEED, the n-byte public seed used to pseudorandomly generate
bitmasks and hash function keys. Although root and SEED formally
would be considered only part of the public key, they are needed e.g.
for signature generation and hence are also required for functions
that do not take the public key as input.

The leaf index idx is initialized to zero when the XMSS private key
is created. The key SK_PRF MUST be sampled from a secure source of
randomness that follows the uniform distribution. The WOTS+ secret
keys MUST be generated as described in Section 3.1. To reduce the
secret key size, a cryptographic pseudorandom method MAY be used as
discussed at the end of this section. SEED is generated as a
uniformly random n-byte string. Although SEED is public, it is
critical for security that it is generated using a good entropy
source. The root node is generated as described below in the section
on key generation (Section 4.1.7). That section also contains an
example algorithm for combined secret and public key generation.

For the following algorithm descriptions, the existence of a method
getWOTS_SK(SK, i) is assumed. This method takes as inputs an XMSS
secret key SK and an integer i and outputs the ith WOTS+ secret key
of SK.

4.1.4. Randomized Tree Hashing

To improve readability we introduce a function RAND_HASH(LEFT, RIGHT,
SEED, ADRS) that does the randomized hashing in the tree. It takes
as input two n-byte values LEFT and RIGHT that represent the left and
the right half of the hash function input, the seed SEED used as key
for PRF and the address ADRS of this hash function call. RAND_HASH
first uses PRF with SEED and ADRS to generate a key KEY and n-byte
bitmasks BM_0, BM_1. Then it returns the randomized hash H(KEY,
(LEFT XOR BM_0) || (RIGHT XOR BM_1)).

Algorithm 7: RAND_HASH

    Input: n-byte value LEFT, n-byte value RIGHT, seed SEED,
            address ADRS
    Output: n-byte randomized hash

    ADRS.setKeyBit(0);
    ADRS.setBlockBit(0);
    BM_0 = PRF(SEED, ADRS);
    ADRS.setBlockBit(1);
    BM_1 = PRF(SEED, ADRS);
    ADRS.setKeyBit(1);
    ADRS.setBlockBit(0);
    KEY = PRF(SEED, ADRS);
    return H(KEY, (LEFT XOR BM_0) || (RIGHT XOR BM_1));

4.1.5. L-Trees

To compute the leaves of the binary hash tree, a so-called L-tree is
used. An L-tree is an unbalanced binary hash tree, distinct but
similar to the main XMSS binary hash tree. The algorithm ltree
(Algorithm 8) takes as input a WOTS+ public key pk and compresses it
to a single n-byte value pk[0]. Towards this end it also takes an
L-tree address ADRS as input that encodes the address of the L-tree,
and the seed SEED.

Algorithm 8: ltree

    Input: WOTS+ public key pk, address ADRS, seed SEED
    Output: n-byte compressed public key value pk[0]

    unsigned int len' = len;
    ADRS.setTreeHeight(0);
    while (len' > 1) {
      for (i = 0; i < floor(len' / 2); i++) {
        ADRS.setTreeIndex(i);
        pk[i] = RAND_HASH(pk[2i], pk[2i + 1], SEED, ADRS);
      }
      if (len' % 2 == 1) {
        pk[floor(len' / 2)] = pk[len' - 1];
      }
      len' = ceil(len' / 2);
      ADRS.setTreeHeight(ADRS.getTreeHeight() + 1);
    }
    return pk[0];

4.1.6. TreeHash

   For the computation of the internal n-byte nodes of a Merkle tree,
   the subroutine treeHash (Algorithm 9) accepts an XMSS secret key SK
   (including seed SEED), an unsigned integer s (the start index), an
   unsigned integer t (the target node height), and an address ADRS that
   encodes the address of the containing tree. For the height of a node
   within a tree counting starts with the leaves at height zero. The
   treeHash algorithm returns the root node of a tree of height t with
   the leftmost leaf being the hash of the WOTS+ pk with index s. It is
   REQUIRED that s % 2^t = 0, i.e. that the leaf at index s is a left
   most leaf of a sub-tree of height t. Otherwise the hash-addressing
   scheme fails. The treeHash algorithm described here uses a stack
   holding up to (t - 1) nodes, with the usual stack functions push()
   and pop(). We furthermore assume that the height of a node (an
   unsigned integer) is stored alongside a node's value (an n-byte
   string) on the stack.

   Algorithm 9: treeHash

     Input: XMSS secret key SK, start index s, target node height t,
            address ADRS
     Output: n-byte root node - top node on Stack

     if(s % (1 << t) != 0) return -1;
     for (i = 0; i < 2^t; i++) {
       SEED = getSEED(SK);
       ADRS.setOTSBit(1);
       ADRS.setOTSAddress(s+i);
       pk = WOTS_genPK (getWOTS_SK(SK, s+i), SEED, ADRS);
       ADRS.setOTSBit(0);
       ADRS.setLTreeBit(1);
       ADRS.setLTreeAddress(s + i);
       node = ltree(pk, SEED, ADRS);
       ADRS.setLTreeBit(0);
       ADRS.setTreeHeight(0);
       ADRS.setTreeIndex(i + s);
       while (Top node on Stack has same height t' as node) {
          ADRS.setTreeIndex((ADRS.getTreeIndex() - 1) / 2);
          node = RAND_HASH(Stack.pop(), node, SEED, ADRS);
          ADRS.setTreeHeight(ADRS.getTreeHeight() + 1);
       }
       Stack.push(node);
     }
     return Stack.pop();

4.1.7. XMSS Key Generation

   The XMSS key pair is computed as described in XMSS_keyGen (Algorithm
   10). The XMSS public key PK consists of the root of the binary hash
   tree and the seed SEED, both also stored in SK. The root is computed
   using treeHash. For XMSS, there is only a single main tree. Hence,
   the used address is set to the all-zero string in the beginning.
   Note that we do not define any specific format or handling for the
   XMSS secret key SK by introducing this algorithm. It relates to
   requirements described earlier and simply shows a basic but very
   inefficient example to initialize a secret key.

   Algorithm 10: XMSS_keyGen - Generate an XMSS key pair

```
     Input: /
     Output: XMSS secret key SK, XMSS public key PK

     // Example initialization for SK-specific contents
     idx = 0;
     for ( i = 0; i < 2^h; i++ ) {
       WOTS_genSK(wots_sk[i]);
     }
     initialize SK_PRF with a uniformly random n-byte string;
     setSK_PRF(SK, SK_PRF);

     // Initialization for common contents
     initialize SEED with a uniformly random n-byte string;
     setSEED(SK, SEED);
     setWOTS_SK(SK, wots_sk));
     ADRS = toByte(0, 32);
     root = treeHash(SK, 0, h, SEED, ADRS);

     SK = idx || wots_sk || SK_PRF || root || SEED;
     PK = root || SEED;
     return (SK || PK);
```

   The above is just an example algorithm. It is strongly RECOMMENDED
   to use pseudorandom key generation to reduce the secret key size.
   Public and private key generation MAY be interleaved to save space.
   Especially, when a pseudorandom method is used to generate the secret
   key, generation MAY be done when the respective WOTS+ key pair is
   needed by treeHash.

   The format of an XMSS public key is given below.

XMSS Public Key

```
          +--------------------------------+
          |          algorithm OID          |
          +--------------------------------+
          |                                |
          |                                |
          |            root node           |       n bytes
          |                                |
          |                                |
          +--------------------------------+
          |                                |
          |                                |
          |              SEED              |       n bytes
          |                                |
          |                                |
          +--------------------------------+
```
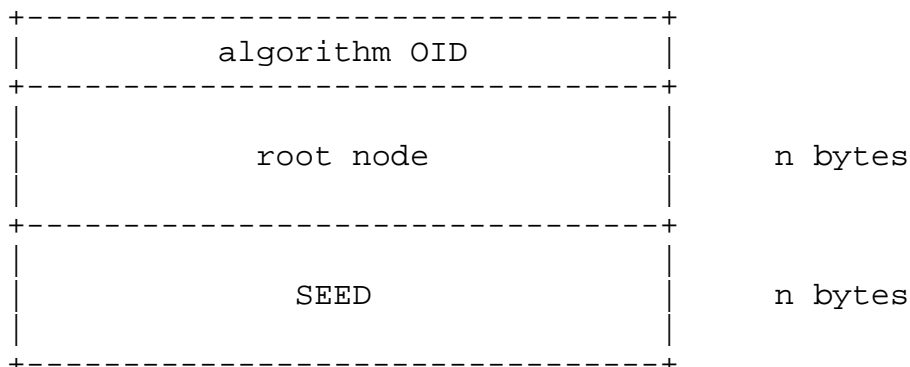
4.1.8. XMSS Signature

   An XMSS signature is a $(4 + n + (len + h) * n)$-byte string consisting
   of

        the index idx_sig of the used WOTS+ key pair (4 bytes),

        a byte string r used for randomized message hashing (n bytes),

        a WOTS+ signature sig_ots (len * n bytes),

        the so-called authentication path 'auth' for the leaf associated
        with the used WOTS+ key pair (h * n bytes).

   The authentication path is an array of h n-byte strings. It contains
   the siblings of the nodes on the path from the used leaf to the root.
   It does not contain the nodes on the path itself. These nodes are
   needed by a verifier to compute a root node for the tree from the
   WOTS+ public key. A node Node is addressed by its position in the
   tree. Node(x, y) denotes the xth node on level y with $x = 0$ being
   the leftmost node on a level. The leaves are on level 0, the root is
   on level h. An authentication path contains exactly one node on
   every layer $0 <= x <= h - 1$. For the ith WOTS+ key pair, counting
   from zero, the jth authentication path node is

        Node(j, floor(i / (2^j)) XOR 1)

   The computation of the authentication path is discussed in
   Section 4.1.9.

   The data format for a signature is given below.

XMSS Signature

```
          +-------------------------------+
          |                               |
          |        index idx_sig          |      4 bytes
          |                               |
          +-------------------------------+
          |                               |
          |        randomness r           |      n bytes
          |                               |
          +-------------------------------+
          |                               |
          |    WOTS+ signature sig_ots    |      len * n bytes
          |                               |
          +-------------------------------+
          |                               |
          |           auth[0]             |      n bytes
          |                               |
          +-------------------------------+
          |                               |
          ~           ....                ~
          |                               |
          +-------------------------------+
          |                               |
          |          auth[h - 1]          |      n bytes
          |                               |
          +-------------------------------+
```
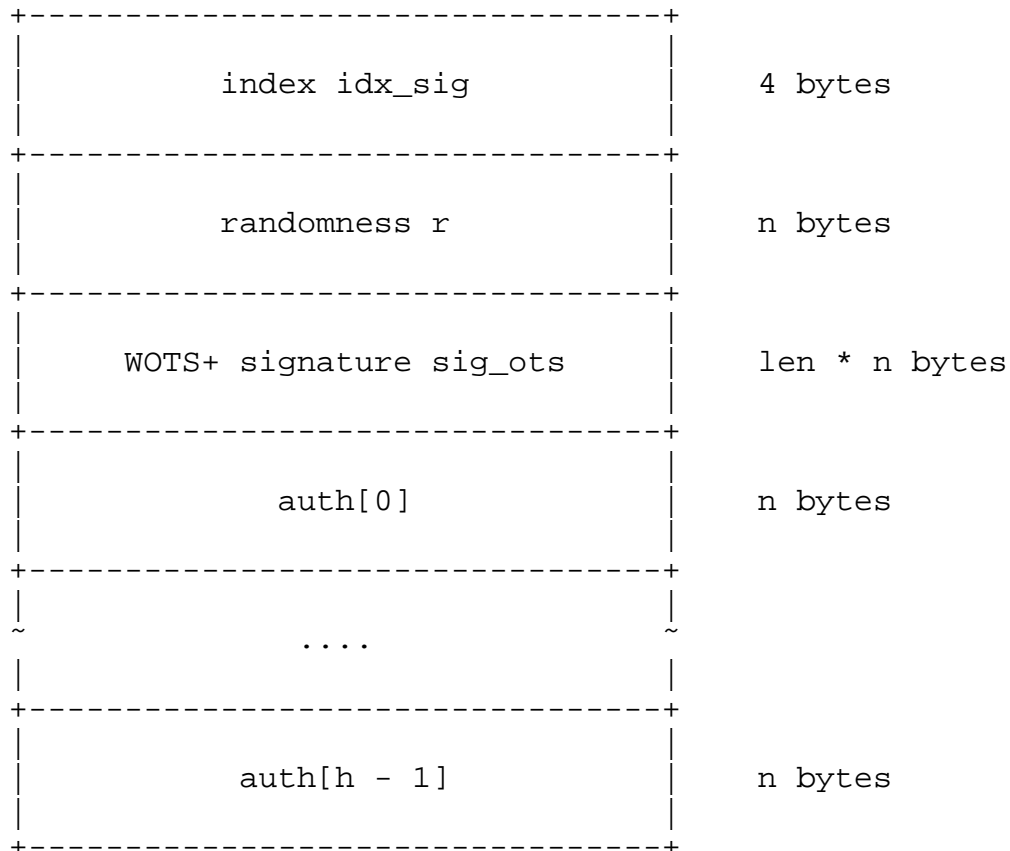
4.1.9. XMSS Signature Generation

   To compute the XMSS signature of a message M with an XMSS private
   key, the signer first computes a randomized message digest using a
   random value r, idx_sig, the index of the WOTS+ key pair to be used,
   and the root value from the public key as key. Then a WOTS+
   signature of the message digest is computed using the next unused
   WOTS+ private key. Next, the authentication path is computed.
   Finally, the secret key is updated, i.e. idx is incremented. An
   implementation MUST NOT output the signature before the updated
   private key.

   The node values of the authentication path MAY be computed in any
   way. This computation is assumed to be performed by the subroutine
   buildAuth for the function XMSS_sign, as below. The fastest
   alternative is to store all tree nodes and set the array in the
   signature by copying the respective nodes. The least storage-
   intensive alternative is to recompute all nodes for each signature
   online using the treeHash algorithm (Algorithm 9). There exist
   several algorithms in between, with different time/storage trade-

offs. For an overview, see [BDS09]. A further approach can be found
in [KMN14]. Note that the details of this procedure are not relevant
to interoperability; it is not necessary to know any of these details
in order to perform the signature verification operation. The
following version of buildAuth is just given for completeness. It is
a simple example for understanding, but extremely inefficient. The
use of one of the alternative algorithms is strongly RECOMMENDED.

Given an XMSS secret key SK, all nodes in a tree are determined.
Their value is defined in terms of treeHash (Algorithm 9). Hence,
one can compute the authentication path as follows:

(Example) buildAuth - Compute the authentication path for the ith
WOTS+ key pair

```
   Input: XMSS secret key SK, WOTS+ key pair index i, ADRS
   Output: Authentication path auth

   for ( j = 0; j < h; j++ ) {
     k = floor(i / (2^j)) XOR 1;
     auth[j] = treeHash(SK, k * 2^j, j, ADRS);
   }
```

We split the description of the signature generation into two main
algorithms. The first one, treeSig (Algorithm 11), generates the
main part of an XMSS signature and is also used by the multi-tree
version XMSS^MT. XMSS_sign (Algorithm 12) calls treeSig but handles
message compression before and the secret key update afterwards.

The algorithm treeSig (Algorithm 11) described below calculates the
WOTS+ signature on an n-byte message and the corresponding
authentication path. treeSig takes as inputs an n-byte message M',
an XMSS secret key SK, and an address ADRS. It returns the
concatenation of the WOTS+ signature sig_ots and authentication path
auth.

Algorithm 11: treeSig - Generate a WOTS+ signature on a message with corresponding authentication path

```
   Input: n-byte message M', XMSS secret key SK, ADRS
   Output: Concatenation of WOTS+ signature sig_ots and
           authentication path auth

   idx_sig = getIdx(SK);
   auth = buildAuth(SK, idx_sig, ADRS);
   ADRS.setOTSBit(1);
   ADRS.setOTSAddress(idx_sig);
   sig_ots = WOTS_sign(getWOTS_SK(SK, idx_sig),
                       M', getSEED(SK), ADRS);
   Sig = (sig_ots || auth);
   return Sig;
```

The algorithm XMSS_sign (Algorithm 12) described below calculates an updated secret key SK and a signature on a message M. XMSS_sign takes as inputs a message M of arbitrary length, and an XMSS secret key SK. It returns the byte string containing the concatenation of the updated secret key SK and the signature Sig.

Algorithm 12: XMSS_sign - Generate an XMSS signature and update the XMSS secret key

```
   Input: Message M, XMSS secret key SK
   Output: Updated SK, XMSS signature Sig

   idx_sig = getIdx(SK);
   ADRS = toByte(0, 32);
   byte[n] r = PRF(getSK_PRF(SK), toByte(idx_sig, 32));
   byte[n] M' = H_msg(r || getRoot(SK) || (toByte(idx_sig, n)), M);
   Sig = (idx_sig || r || treeSig(M', SK, ADRS));
   setIdx(SK, idx_sig + 1);
   return (SK || Sig);
```

4.1.10. XMSS Signature Verification

An XMSS signature is verified by first computing the message digest using randomness r, index idx_sig, the root from PK and message M. Then the used WOTS+ public key pk_ots is computed from the WOTS+ signature using WOTS_pkFromSig. The WOTS+ public key in turn is used to compute the corresponding leaf using an L-tree. The leaf, together with index idx_sig and authentication path auth is used to compute an alternative root value for the tree. The verification succeeds if and only if the computed root value matches the one in the XMSS public key. In any other case it MUST return fail.

As for signature generation, we split verification into two parts to
allow for reuse in the XMSS^MT description. The steps also needed
for XMSS^MT are done by the function XMSS_rootFromSig (Algorithm 13).
XMSS_verify (Algorithm 14) calls XMSS_rootFromSig as a subroutine and
handles the XMSS-specific steps.

The main part of XMSS signature verification is done by the function
XMSS_rootFromSig (Algorithm 13) described below. XMSS_rootFromSig
takes as inputs an index idx_sig, a WOTS+ signature sig_ots, an
authentication path auth, an n-byte message M', seed SEED, and
address ADRS. XMSS_rootFromSig returns an n-byte string holding the
value of the root of a tree defined by the input data.

Algorithm 13: XMSS_rootFromSig - Compute a root node from a tree
signature

```
  Input: index idx_sig, WOTS+ signature sig_ots, authentication path
         auth, n-byte message M', seed SEED, address ADRS
  Output: n-byte root value node[0]

  ADRS.setOTSBit(1);
  ADRS.setOTSAddress(idx_sig);
  pk_ots = WOTS_pkFromSig(sig_ots, M', SEED, ADRS);
  ADRS.setOTSBit(0);
  ADRS.setLTreeBit(1);
  ADRS.setLTreeAddress(idx_sig);
  byte[n][2] node;
  node[0] = ltree(pk_ots, SEED, ADRS);
  ADRS.setLTreeBit(0);
  ADRS.setTreeIndex(idx_sig);
  for ( k = 0; k < h; k++ ) {
    ADRS.setTreeHeight(k);
    if ( (floor(idx_sig / (2^k)) % 2) == 0 ) {
      ADRS.setTreeIndex(ADRS.getTreeIndex() / 2);
      node[1] = RAND_HASH(node[0], auth[k], SEED, ADRS);
    } else {
      ADRS.setTreeIndex(ADRS.getTreeIndex() - 1 / 2);
      node[1] = RAND_HASH(auth[k], node[0], SEED, ADRS);
    }
    node[0] = node[1];
  }
  return node[0];
```

The full XMSS signature verification is depicted below (Algorithm
14). It handles message compression, delegates the root computation
to XMSS_rootFromSig, and compares the result to the value in the
public key. XMSS_verify takes an XMSS signature Sig, a message M,
and an XMSS public key PK. XMSS_verify returns true if and only if

Sig is a valid signature on M under public key PK. Otherwise, it
returns false.

Algorithm 14: XMSS_verify - Verify an XMSS signature using the
corresponding XMSS public key and a message

```
Input: XMSS signature Sig, message M, XMSS public key PK
Output: Boolean

ADRS = toByte(0, 32);
byte[n] M' = H_msg(r || getRoot(PK) || (toByte(idx_sig, n)), M);

byte[n] node = XMSS_rootFromSig(idx_sig, sig_ots, auth, M',
                               getSEED(PK), ADRS);
if ( node == getRoot(PK) ) {
  return true;
} else {
  return false;
}
```

4.1.11. Pseudorandom Key Generation

An implementation MAY use a cryptographically secure pseudorandom
method to generate the XMSS secret key from a single n-byte value.
For example, the method suggested in [BDH11] and explained below MAY
be used. Other methods MAY be used. The choice of a pseudorandom
method does not affect interoperability, but the cryptographic
strength MUST match that of the used XMSS parameters.

For XMSS a similar method than the one used for WOTS+ can be used.
The suggested method from [BDH11] can be described using PRF. During
key generation a uniformly random n-byte string S is sampled from a
secure source of randomness. This seed S MUST NOT be confused with
the public seed SEED. The seed S MUST be independent of SEED and as
it is the main secret, it MUST be kept secret. This seed S is used
to generate an n-byte value S_ots for each WOTS+ key pair. The
n-byte value S_ots can then be used to compute the respective WOTS+
secret key using the method described in Section 3.1.7. The seeds
for the WOTS+ key pairs are computed as S_ots[i] = PRF(S, toByte(i,
32)) where i is the index of the WOTS+ key pair. An advantage of
this method is that a WOTS+ key can be computed using only len + 1
evaluations of PRF when S is given.

4.1.12. Free Index Handling and Partial Secret Keys

Some applications might require to work with partial secret keys or
copies of secret keys. Examples include delegation of signing rights
/ proxy signatures, and load balancing. Such applications MAY use

their own key format and MAY use a signing algorithm different from
the one described above. The index in partial secret keys or copies
of a secret key MAY be manipulated as required by the applications.
However, applications MUST establish means that guarantee that each
index and thereby each WOTS+ key pair is used to sign only a single
message.

4.2. XMSS^MT: Multi-Tree XMSS

XMSS^MT is a method for signing a large but fixed number of messages.
It was first described in [HRB13]. It builds on XMSS. XMSS^MT uses
a tree of several layers of XMSS trees, a so-called hypertree. The
trees on top and intermediate layers are used to sign the root nodes
of the trees on the respective layer below. Trees on the lowest
layer are used to sign the actual messages. All XMSS trees have
equal height.

Consider an XMSS^MT tree of total height h that has d layers of XMSS
trees of height h / d. Then layer d - 1 contains one XMSS tree,
layer d - 2 contains 2^(h / d) XMSS trees, and so on. Finally, layer
0 contains 2^(h - h / d) XMSS trees.

4.2.1. XMSS^MT Parameters

In addition to all XMSS parameters, an XMSS^MT system requires the
number of tree layers d, specified as an integer value that divides h
without remainder. The same tree height h / d and the same
Winternitz parameter w are used for all tree layers.

All the trees on higher layers sign root nodes of other trees which
are n-byte strings. Hence, no message compression is needed and
WOTS+ is used to sign the root nodes themselves instead of their hash
values.

4.2.2. XMSS^MT Key generation

An XMSS^MT private key SK_MT consists of one reduced XMSS private key
for each XMSS tree. These reduced XMSS private keys just contain the
WOTS+ secret keys corresponding to that XMSS key pair and no
pseudorandom function key, no index, no public seed, no root node.
Instead, SK_MT contains a single n-byte pseudorandom function key
SK_PRF, a single (ceil(h / 8))-byte index idx_MT, a single n-byte
seed SEED, and a single root value root which is the root of the
single tree on the top layer. The index is a global index over all
WOTS+ key pairs of all XMSS trees on layer 0. It is initialized with
0. It stores the index of the last used WOTS+ key pair on the bottom
layer, i.e. a number between 0 and 2^h - 1.

The reduced XMSS secret keys MUST either be generated as described in Section 4.1.3 or using a cryptographic pseudorandom method as discussed at the end of this section. As for XMSS, the PRF key SK_PRF MUST be sampled from a secure source of randomness that follows the uniform distribution. SEED is generated as a uniformly random n-byte string. Although SEED is public, it is critical for security that it is generated using a good entropy source. The root is the root node of the single XMSS tree on the top layer. Its computation is explained below. As for XMSS, root and SEED are public information and would classically be considered part of the public key. However, as both are needed for signing, which only takes the secret key, they are also part of SK_MT.

This document does not define any specific format for the XMSS^MT secret key SK_MT as it is not required for interoperability. The algorithm descriptions below use a function getXMSS_SK(SK, x, y) that outputs the reduced secret key of the xth XMSS tree on the yth layer.

The XMSS^MT public key PK_MT contains the root of the single XMSS tree on layer d - 1 and the seed SEED. These are the same values as in the secret key SK_MT. The pseudorandom function PRF keyed with SEED is used to generate the bitmasks and keys for all XMSS trees. XMSSMT_keyGen (Algorithm 15) shows example pseudocode to generate SK_MT and PK_MT. The n-byte root node of the top layer tree is computed using treeHash. The algorithm XMSSMT_keyGen outputs an XMSS^MT secret key SK_MT and an XMSS^MT public key PK_MT. The algorithm below gives an example of how the reduced XMSS secret keys can be generated. However, any of the above mentioned ways is acceptable as long as the cryptographic strength of the used method matches or superseeds that of the used XMSS^MT parameter set.

Algorithm 15: XMSSMT_keyGen - Generate an XMSS^MT key pair

```
Input: /
Output: XMSS^MT secret key SK_MT, XMSS^MT public key PK_MT

// Example initialization
idx_MT = 0;
setIdx(SK_MT, idx_MT);
initialize SK_PRF with a uniformly random n-byte string;
setSK_PRF(SK_MT, SK_PRF);
initialize SEED with a uniformly random n-byte string;
setSEED(SK_MT, SEED);

// generate reduced XMSS secret keys
ADRS = toByte(0, 32);
for ( layer = 0; layer < d; layer++ ) {
   ADRS.setLayerAddress(layer);
   for ( tree = 0; tree <
         (1 << ((d - 1 - layer) * (h / d)));
         tree++ ) {
      ADRS.setTreeAddress(tree);
      for ( i = 0; i < 2^h; i++ ) {
         WOTS_genSK(wots_sk[i]);
      }
      setXMSS_SK(SK_MT, wots_sk, tree, layer);
   }
}

SK = getXMSS_SK(SK_MT, 0, d - 1);
setSEED(SK, SEED);
root = treeHash(SK, 0, h / d, ADRS);
setRoot(SK_MT, root);

PK_MT = (root || SEED);
return (SK_MT || PK_MT);
```

The above is just an example algorithm. It is strongly RECOMMENDED
to use pseudorandom key generation to reduce the secret key size.
Public and private key generation MAY be interleaved to save space.
Especially, when a pseudorandom method is used to generate the secret
key, generation MAY be delayed to the point when the respective WOTS+
key pair is needed by another algorithm.

The format of an XMSS^MT public key is given below.

XMSS^MT Public Key

```
            +-------------------------------+
            |          algorithm OID        |
            +-------------------------------+
            |                               |
            |                               |
            |           root node           |        n bytes
            |                               |
            |                               |
            +-------------------------------+
            |                               |
            |                               |
            |             SEED              |        n bytes
            |                               |
            |                               |
            +-------------------------------+
```
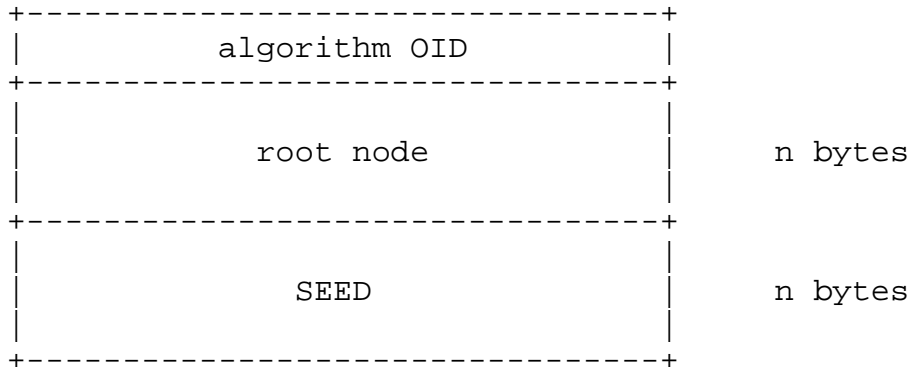
4.2.3. XMSS^MT Signature

   An XMSS^MT signature Sig_MT is a byte string of length (ceil(h / 8) +
   n + (h + d * len) * n). It consists of

      the index idx_sig of the used WOTS+ key pair on the bottom layer
      (ceil(h / 8) bytes),

      a byte string r used for randomized message hashing (n bytes),

      d reduced XMSS signatures ((h / d + len) * n bytes each).

   The reduced XMSS signatures only contain a WOTS+ signature sig_ots
   and an authentication path auth. They contain no index idx and no
   byte string r.

   The data format for a signature is given below.

XMSS^MT signature

```
        +-------------------------------+
        |                               |
        |         index idx_sig         |   ceil(h / 8) bytes
        |                               |
        +-------------------------------+
        |                               |
        |         randomness r          |   n bytes
        |                               |
        +-------------------------------+
        |                               |
        |  (reduced) XMSS signature Sig |   (h / d + len) * n bytes
        |        (bottom layer 0)       |
        |                               |
        +-------------------------------+
        |                               |
        |  (reduced) XMSS signature Sig |   (h / d + len) * n bytes
        |           (layer 1)           |
        |                               |
        +-------------------------------+
        |                               |
        ~            ....               ~
        |                               |
        +-------------------------------+
        |                               |
        |  (reduced) XMSS signature Sig |   (h / d + len) * n bytes
        |         (layer d - 1)         |
        |                               |
        +-------------------------------+
```
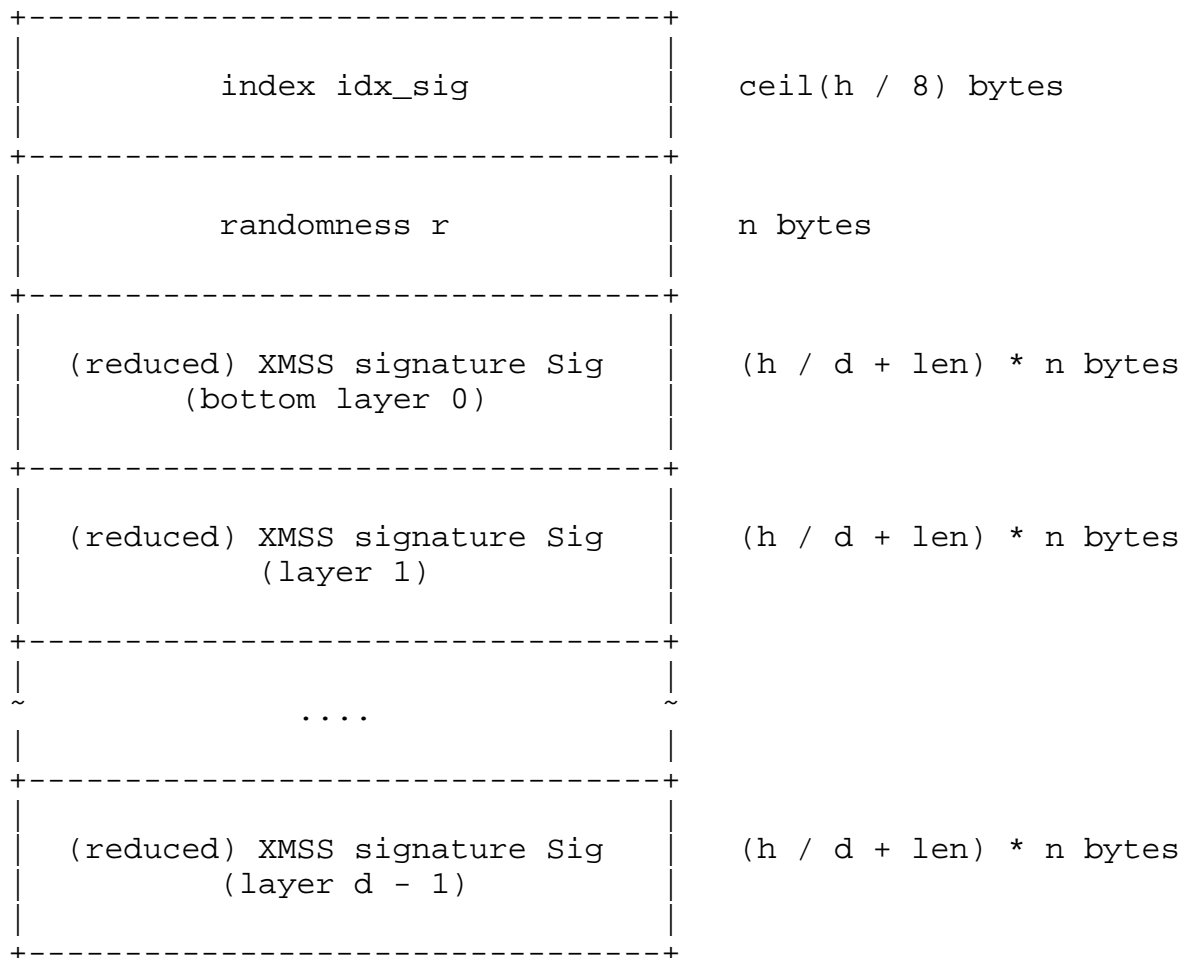
4.2.4. XMSS^MT Signature Generation

To compute the XMSS^MT signature Sig_MT of a message M using an
XMSS^MT private key SK_MT, XMSSMT_sign (Algorithm 16) described below
uses treeSig as defined in Section 4.1.9. First, the signature index
is set to idx_sig. Next, PRF is used to compute a pseudorandom
n-byte string r. This n-byte string, idx_sig, and the root node from
PK_MT are then used to compute a randomized message digest of length
n. The message digest is signed using the WOTS+ key pair on the
bottom layer with absolute index idx. The authentication path for
the WOTS+ key pair is computed as well as the root of the containing
XMSS tree. The root is signed by the parent XMSS tree. This is
repeated until the top tree is reached.

Algorithm 16: XMSSMT_sign - Generate an XMSS^MT signature and update
the XMSS^MT secret key

```
   Input: Message M, XMSS^MT secret key SK_MT
   Output: Updated SK_MT, signature Sig_MT

   // Init
   ADRS = toByte(0, 32);
   SEED = getSEED(SK_MT);
   SK_PRF = getSK_PRF(SK_MT);
   idx_sig = getIdx(SK_MT);

   // Update SK_MT
   setIdx(SK_MT, idx_sig + 1);

   // Message compression
   byte[n] r = PRF(SK_PRF, toByte(idx_sig, 32));
   byte[n] M' = H_msg(r || getRoot(SK_MT) || (toByte(idx_sig, n)), M);

   // Sign
   Sig_MT = idx_sig;
   unsigned int idx_tree
               = (h - h / d) most significant bits of idx_sig;
   unsigned int idx_leaf = (h / d) least significant bits of idx_sig;
   SK = idx_leaf || getXMSS_SK(SK_MT, idx_tree, 0) ||SK_PRF
          || toByte(0, n) || SEED;
   ADRS.setLayerAddress(0);
   ADRS.setTreeAddress(idx_tree);
   Sig_tmp = treeSig(M', SK, ADRS);
   Sig_MT = Sig_MT || r || Sig_tmp;
   for ( j = 1; j < d; j++ ) {
      root = treeHash(SK, 0, h / d, ADRS);
      idx_leaf = (h / d) least significant bits of idx_tree;
      idx_tree = (h - j * (h / d)) most significant bits of idx_tree;
      SK = idx_leaf || getXMSS_SK(SK_MT, idx_tree, j) || SK_PRF
             || toByte(0, n) || SEED;
      ADRS.setLayerAddress(j);
      ADRS.setTreeAddress(idx_tree);
      Sig_tmp = treeSig(root, SK, ADRS);
      Sig_MT = Sig_MT || Sig_tmp;
   }
   return SK_MT || Sig_MT;
```

Algorithm 16 is only one method to compute XMSS^MT signatures.
Especially, there exist time-memory trade-offs that allow to reduce
the signing time to less than the signing time of an XMSS scheme with
tree height h / d. These trade-offs prevent certain values from
being recomputed several times by keeping a state and distribute all

computations over all signature generations. Details can be found in [Huelsing13a].

4.2.5. XMSS^MT Signature Verification

XMSS^MT signature verification (Algorithm 17) can be summarized as d XMSS signature verifications with small changes. First, the message is hashed. The XMSS signatures are then all on n-byte values. Second, instead of comparing the computed root node to a given value, a signature on this root node is verified. Only the root node of the top tree is compared to the value in the XMSS^MT public key. XMSSMT_verify uses XMSS_rootFromSig. The function getXMSSSignature(Sig_MT, i) returns the ith reduced XMSS signature from the XMSS^MT signature Sig_MT. XMSSMT_verify takes as inputs an XMSS^MT signature Sig_MT, a message M and a public key PK_MT. XMSSMT_verify returns true if and only if Sig_MT is a valid signature on M under public key PK_MT. Otherwise, it returns false.

```
   Algorithm 17: XMSSMT_verify - Verify an XMSS^MT signature Sig_MT on a
   message M using an XMSS^MT public key PK_MT

      Input: XMSS^MT signature Sig_MT, message M,
             XMSS^MT public key PK_MT
      Output: Boolean

      idx_sig = getIdx(Sig_MT);
      SEED = getSEED(PK_MT);
      ADRS = toByte(0, 32);

      byte[n] M' = H_msg(getR(Sig_MT) || getRoot(PK_MT)
                          || (toByte(idx_sig, n)), M);

      unsigned int idx_leaf
                    = (h / d) least significant bits of idx_sig;
      unsigned int idx_tree
                    = (h - h / d) most significant bits of idx_sig;
      Sig' = getXMSSSignature(Sig_MT, 0);
      ADRS.setLayerAddress(0);
      ADRS.setTreeAddress(idx_tree);
      byte[n] node = XMSS_rootFromSig(idx_leaf, getSig_ots(Sig'),
                                        getAuth(Sig'),M', SEED, ADRS);
      for ( j = 1; j < d; j++ ) {
         idx_leaf = (h / d) least significant bits of idx_tree;
         idx_tree = (h - j * h / d) most significant bits of idx_tree;
         Sig' = getXMSSSignature(Sig_MT, j);
         ADRS.setLayerAddress(j);
         ADRS.setTreeAddress(idx_tree);
         node = XMSS_rootFromSig(idx_leaf, getSig_ots(Sig'),
                                 getAuth(Sig'), node, SEED, ADRS);
      }
      if ( node == getRoot(PK_MT) ) {
        return true;
      } else {
        return false;
      }
```

4.2.6. Pseudorandom Key Generation

   Like for XMSS, an implementation MAY use a cryptographically secure
   pseudorandom method to generate the XMSS^MT secret key from a single
   n-byte value. For example, the method explained below MAY be used.
   Other methods MAY be used, too. The choice of a pseudorandom method
   does not affect interoperability, but the cryptographic strength MUST
   match that of the used XMSS^MT parameters.

For XMSS^MT a method similar to that for XMSS and WOTS+ can be used. The method uses PRF. During key generation a uniformly random n-byte string S_MT is sampled from a secure source of randomness. This seed S_MT is used to generate one n-byte value S for each XMSS key pair. This n-byte value can be used to compute the respective XMSS secret key using the method described in Section 4.1.11. Let S[x][y] be the seed for the xth XMSS secret key on layer y. The seeds are computed as S[x][y] = PRF(PRF(S, toByte(y, 32)), toByte(x, 32)).

4.2.7. Free Index Handling and Partial Secret Keys

The content of Section 4.1.12 also applies to XMSS^MT.

5. Parameter Sets

This section provides a basic set of parameter sets which are assumed to cover most relevant applications. Parameter sets for two classical security levels are defined. Parameters with n = 32 provide a classical security level of 256 bits. Parameters with n = 64 provide a classical security level of 512 bits. Considering quantum-computer-aided attacks, these output sizes yield post-quantum security of 128 and 256 bits, respectively.

For the n = 32 and n = 64 settings, we give parameters that use SHA2-256, SHA2-512 as defined in [FIPS180], and SHAKE-128, SHAKE-256 as defined in [FIPS202]. The parameter sets using SHA2-256 are mandatory for deployment and therefore MUST be provided by any implementation. The remaining parameter sets specified in this document are OPTIONAL.

SHA2 does not provide a keyed-mode itself. To implement the keyed hash functions the following is used for SHA2 with n = 32:

    F: SHA2-256(toByte(0, 32) || KEY || M),

    H: SHA2-256(toByte(1, 32) || KEY || M),

    H_msg: SHA2-256(toByte(2, 32) || KEY || M),

    PRF: SHA2-256(toByte(3, 32) || KEY || M).

Accordingly, for SHA2 with n = 64 we use:

    F: SHA2-512(toByte(0, 64) || KEY || M),

    H: SHA2-512(toByte(1, 64) || KEY || M),

    H_msg: SHA2-512(toByte(2, 64) || KEY || M),

      PRF: SHA2-512(toByte(3, 64) || KEY || M).

   The n-byte padding is used for two reasons. First, it is necessary
   that the internal compression function takes 2n-byte blocks but keys
   are n and 3n bytes long. Second, the padding is used to achieve
   independence of the different function families. Finally, for the
   PRF no full-fledged HMAC is needed as the message length is fixed.
   For that reason the simpler construction above suffices.

   Similar constructions are used with SHA3. To implement the keyed
   hash functions the following is used for SHA3 with n = 32:

      F: SHAKE128(toByte(0, 32) || KEY || M, 256),

      H: SHAKE128(toByte(1, 32) || KEY || M, 256),

      H_msg: SHAKE128(toByte(2, 32) || KEY || M, 256),

      PRF: SHAKE128(toByte(3, 32) || KEY || M, 256).

   Accordingly, for SHA3 with n = 64 we use:

      F: SHAKE256(toByte(0, 64) || KEY || M, 512),

      H: SHAKE256(toByte(1, 64) || KEY || M, 512),

      H_msg: SHAKE256(toByte(2, 64) || KEY || M, 512),

      PRF: SHAKE256(toByte(3, 64) || KEY || M, 512).

   We use n-bytes for domain separation for consistency with the SHA2
   implementations.

5.1. WOTS+ Parameters

   To fully describe a WOTS+ signature method, the parameters n, and w,
   as well as the functions F and PRF MUST be specified. This section
   defines several WOTS+ signature systems, each of which is identified
   by a name. Values for len are provided for convenience.

Name	F / PRF	n	w	len
REQUIRED:				
WOTSP_SHA2-256_W16	SHA2-256	32	16	67
OPTIONAL:				
WOTSP_SHA2-512_W16	SHA2-512	64	16	131
WOTSP_SHAKE128_W16	SHAKE128	32	16	67
WOTSP_SHAKE256_W16	SHAKE256	64	16	131

Table 1

The implementation of the single functions is done as described above. XDR formats for WOTS+ are listed in Appendix A.

5.2. XMSS Parameters

To fully describe an XMSS signature method, the parameters n, w, and h, as well as the functions F, H, H_msg, and PRF MUST be specified. This section defines different XMSS signature systems, each of which is identified by a name. We define parameter sets that implement the functions using SHA2 for n = 32 and n = 64 as described above.

Name	Functions	n	w	len	h
REQUIRED:					
XMSS_SHA2-256_W16_H10	SHA2-256	32	16	67	10
XMSS_SHA2-256_W16_H16	SHA2-256	32	16	67	16
XMSS_SHA2-256_W16_H20	SHA2-256	32	16	67	20
OPTIONAL:					
XMSS_SHA2-512_W16_H10	SHA2-512	64	16	131	10
XMSS_SHA2-512_W16_H16	SHA2-512	64	16	131	16
XMSS_SHA2-512_W16_H20	SHA2-512	64	16	131	20
XMSS_SHAKE128_W16_H10	SHAKE128	32	16	67	10
XMSS_SHAKE128_W16_H16	SHAKE128	32	16	67	16
XMSS_SHAKE128_W16_H20	SHAKE128	32	16	67	20
XMSS_SHAKE256_W16_H10	SHAKE256	64	16	131	10
XMSS_SHAKE256_W16_H16	SHAKE256	64	16	131	16
XMSS_SHAKE256_W16_H20	SHAKE256	64	16	131	20

Table 2

The XDR formats for XMSS are listed in Appendix B.

5.3. XMSS^MT Parameters

To fully describe an XMSS^MT signature method, the parameters n, w, h, and d, as well as the functions F, H, H_msg, and PRF MUST be specified. This section defines several XMSS^MT signature systems, each of which is identified by a name. We define parameter sets that implement the functions using SHA2 for n = 32 and n = 64 as described above.

Name	Functions	n	w	len	h	d

REQUIRED:						
XMSSMT_SHA2-256_W16_H20_D2	SHA2-256	32	16	67	20	2
XMSSMT_SHA2-256_W16_H20_D4	SHA2-256	32	16	67	20	4
XMSSMT_SHA2-256_W16_H40_D2	SHA2-256	32	16	67	40	2
XMSSMT_SHA2-256_W16_H40_D4	SHA2-256	32	16	67	40	4
XMSSMT_SHA2-256_W16_H40_D8	SHA2-256	32	16	67	40	8
XMSSMT_SHA2-256_W16_H60_D3	SHA2-256	32	16	67	60	3
XMSSMT_SHA2-256_W16_H60_D6	SHA2-256	32	16	67	60	6
XMSSMT_SHA2-256_W16_H60_D12	SHA2-256	32	16	67	60	12
OPTIONAL:						
XMSSMT_SHA2-512_W16_H20_D2	SHA2-512	64	16	131	20	2
XMSSMT_SHA2-512_W16_H20_D4	SHA2-512	64	16	131	20	4
XMSSMT_SHA2-512_W16_H40_D2	SHA2-512	64	16	131	40	2
XMSSMT_SHA2-512_W16_H40_D4	SHA2-512	64	16	131	40	4
XMSSMT_SHA2-512_W16_H40_D8	SHA2-512	64	16	131	40	8
XMSSMT_SHA2-512_W16_H60_D3	SHA2-512	64	16	131	60	3
XMSSMT_SHA2-512_W16_H60_D6	SHA2-512	64	16	131	60	6
XMSSMT_SHA2-512_W16_H60_D12	SHA2-512	64	16	131	60	12
XMSSMT_SHAKE128_W16_H20_D2	SHAKE128	32	16	67	20	2
XMSSMT_SHAKE128_W16_H20_D4	SHAKE128	32	16	67	20	4
XMSSMT_SHAKE128_W16_H40_D2	SHAKE128	32	16	67	40	2
XMSSMT_SHAKE128_W16_H40_D4	SHAKE128	32	16	67	40	4
XMSSMT_SHAKE128_W16_H40_D8	SHAKE128	32	16	67	40	8
XMSSMT_SHAKE128_W16_H60_D3	SHAKE128	32	16	67	60	3

XMSSMT_SHAKE128_W16_H60_D6	SHAKE128	32	16	67	60	6
XMSSMT_SHAKE128_W16_H60_D12	SHAKE128	32	16	67	60	12
XMSSMT_SHAKE256_W16_H20_D2	SHAKE256	64	16	131	20	2
XMSSMT_SHAKE256_W16_H20_D4	SHAKE256	64	16	131	20	4
XMSSMT_SHAKE256_W16_H40_D2	SHAKE256	64	16	131	40	2
XMSSMT_SHAKE256_W16_H40_D4	SHAKE256	64	16	131	40	4
XMSSMT_SHAKE256_W16_H40_D8	SHAKE256	64	16	131	40	8
XMSSMT_SHAKE256_W16_H60_D3	SHAKE256	64	16	131	60	3
XMSSMT_SHAKE256_W16_H60_D6	SHAKE256	64	16	131	60	6
XMSSMT_SHAKE256_W16_H60_D12	SHAKE256	64	16	131	60	12

                              Table 3

   XDR formats for XMSS^MT are listed in Appendix C.

6. Rationale

   The goal of this note is to describe the WOTS+, XMSS and XMSS^MT
   algorithms following the scientific literature. Other signature
   methods are out of scope and may be an interesting follow-on work.
   The description is done in a modular way that allows to base a
   description of stateless hash-based signature algorithms like SPHINCS
   [BHH15] on it.

   The draft slightly deviates from the scientific literature using a
   tweak that prevents multi-user / multi-target attacks against H_msg.
   To this end, the public key as well as the index of the used one-time
   key pair become part of the hash function key. Thereby we achieve a
   domain separation that forces an attacker to decide which hash value
   to attack.

   For the generation of the randomness used for randomized message
   hashing, we apply a PRF, keyed with a secret value, to the index of
   the used one-time key pair instead of the message. The reason is
   that this requires to process the message only once instead of twice.
   For long messages this improves speed and simplifies implementations
   on resource constrained devices that cannot hold the entire message
   in storage.

We give one mandatory set of parameters using SHA2-256. The reasons are twofold. On the one hand, SHA2-256 is available on most platforms today and part of most cryptographic libraries. On the other hand, a 256-bit hash function leads parameters that provides 128 bit of security even against quantum-computer-aided attacks. A post-quantum security level of 256 bit seems overly conservative. However, to prepare for possible cryptanalytic breakthroughs, we also provide OPTIONAL parameter sets using the less common SHA2-512, SHAKE-256, and SHAKE-512 functions.

We suggest the value w = 16 for the Winternitz parameter. No bigger values are included since the decrease in signature size then becomes less significant. Furthermore, the value w = 16 considerably simplifies the implementations of some of the algorithms. Please note that we do allow w = 4, but limit the specified parameter sets to w = 16 for efficiency reasons.

The signature and public key formats are designed so that they are easy to parse. Each format starts with a 32-bit enumeration value that indicates all of the details of the signature algorithm and hence defines all of the information that is needed in order to parse the format.

The enumeration values used in this note are palindromes, which have the same byte representation in either host order or network order. This fact allows an implementation to omit the conversion between byte order for those enumerations. Note however that the idx field used in XMSS and XMSS^MT signatures and secret keys MUST be properly converted to and from network byte order; this is the only field that requires such conversion. There are 2^{32} XDR enumeration values, 2^{16} of which are palindromes, which is adequate for the foreseeable future. If there is a need for more assignments, non-palindromes can be assigned.

7. IANA Considerations

The Internet Assigned Numbers Authority (IANA) is requested to create three registries: one for WOTS+ signatures as defined in Section 3, one for XMSS signatures and one for XMSS^MT signatures; the latter two being defined in Section 4. For the sake of clarity and convenience, the first sets of WOTS+, XMSS, and XMSS^MT parameter sets are defined in Section 5. Additions to these registries require that a specification be documented in an RFC or another permanent and readily available reference in sufficient details to make interoperability between independent implementations possible. Each entry in the registry contains the following elements:

   a short name, such as "XMSS_SHA2-256_W16_H20",

a positive number, and

a reference to a specification that completely defines the
signature method test cases that can be used to verify the
correctness of an implementation.

Requests to add an entry to the registry MUST include the name and
the reference. The number is assigned by IANA. These number
assignments SHOULD use the smallest available palindromic number.
Submitters SHOULD have their requests reviewed by the IRTF Crypto
Forum Research Group (CFRG) at cfrg@ietf.org. Interested applicants
that are unfamiliar with IANA processes should visit
http://www.iana.org.

The numbers between 0xDDDDDDDD (decimal 3,722,304,989) and 0xFFFFFFFF
(decimal 4,294,967,295) inclusive, will not be assigned by IANA, and
are reserved for private use; no attempt will be made to prevent
multiple sites from using the same value in different (and
incompatible) ways [RFC2434].

The WOTS+ registry is as follows.

```
+---------------------+------------+--------------------+
| Name                | Reference  | Numeric Identifier |
+---------------------+------------+--------------------+
| WOTSP_SHA2-256_W16  | Section 5.1 |     0x01000001     |
|                     |            |                    |
| WOTSP_SHA2-512_W16  | Section 5.1 |     0x02000002     |
|                     |            |                    |
| WOTSP_SHAKE128_W16  | Section 5.1 |     0x03000003     |
|                     |            |                    |
| WOTSP_SHAKE256_W16  | Section 5.1 |     0x04000004     |
+---------------------+------------+--------------------+
```

Table 4

The XMSS registry is as follows.

Name	Reference	Numeric Identifier
XMSS_SHA2-256_W16_H10	Section 5.2	0x01000001
XMSS_SHA2-256_W16_H16	Section 5.2	0x02000002
XMSS_SHA2-256_W16_H20	Section 5.2	0x03000003
XMSS_SHA2-512_W16_H10	Section 5.2	0x04000004
XMSS_SHA2-512_W16_H16	Section 5.2	0x05000005
XMSS_SHA2-512_W16_H20	Section 5.2	0x06000006
XMSS_SHAKE128_W16_H10	Section 5.2	0x07000007
XMSS_SHAKE128_W16_H16	Section 5.2	0x08000008
XMSS_SHAKE128_W16_H20	Section 5.2	0x09000009
XMSS_SHAKE256_W16_H10	Section 5.2	0x0a00000a
XMSS_SHAKE256_W16_H16	Section 5.2	0x0b00000b
XMSS_SHAKE256_W16_H20	Section 5.2	0x0c00000c

Table 5

The XMSS^MT registry is as follows.

Name	Reference	Numeric Identifier
XMSSMT_SHA2-256_W16_H20_D2	Section 5.3	0x01000001
XMSSMT_SHA2-256_W16_H20_D4	Section 5.3	0x02000002
XMSSMT_SHA2-256_W16_H40_D2	Section 5.3	0x03000003
XMSSMT_SHA2-256_W16_H40_D4	Section 5.3	0x04000004
XMSSMT_SHA2-256_W16_H40_D8	Section 5.3	0x05000005
XMSSMT_SHA2-256_W16_H60_D3	Section 5.3	0x06000006
XMSSMT_SHA2-256_W16_H60_D6	Section 5.3	0x07000007

XMSSMT_SHA2-256_W16_H60_D12	Section 5.3	0x08000008	
XMSSMT_SHA2-512_W16_H20_D2	Section 5.3	0x09000009	
XMSSMT_SHA2-512_W16_H20_D4	Section 5.3	0x0a00000a	
XMSSMT_SHA2-512_W16_H40_D2	Section 5.3	0x0b00000b	
XMSSMT_SHA2-512_W16_H40_D4	Section 5.3	0x0c00000c	
XMSSMT_SHA2-512_W16_H40_D8	Section 5.3	0x0d00000d	
XMSSMT_SHA2-512_W16_H60_D3	Section 5.3	0x0e00000e	
XMSSMT_SHA2-512_W16_H60_D6	Section 5.3	0x0f00000f	
XMSSMT_SHA2-512_W16_H60_D12	Section 5.3	0x01010101	
XMSSMT_SHAKE128_W16_H20_D2	Section 5.3	0x02010102	
XMSSMT_SHAKE128_W16_H20_D4	Section 5.3	0x03010103	
XMSSMT_SHAKE128_W16_H40_D2	Section 5.3	0x04010104	
XMSSMT_SHAKE128_W16_H40_D4	Section 5.3	0x05010105	
XMSSMT_SHAKE128_W16_H40_D8	Section 5.3	0x06010106	
XMSSMT_SHAKE128_W16_H60_D3	Section 5.3	0x07010107	
XMSSMT_SHAKE128_W16_H60_D6	Section 5.3	0x08010108	
XMSSMT_SHAKE128_W16_H60_D12	Section 5.3	0x09010109	
XMSSMT_SHAKE256_W16_H20_D2	Section 5.3	0x0a01010a	
XMSSMT_SHAKE256_W16_H20_D4	Section 5.3	0x0b01010b	
XMSSMT_SHAKE256_W16_H40_D2	Section 5.3	0x0c01010c	
XMSSMT_SHAKE256_W16_H40_D4	Section 5.3	0x0d01010d	
XMSSMT_SHAKE256_W16_H40_D8	Section 5.3	0x0e01010e	
XMSSMT_SHAKE256_W16_H60_D3	Section 5.3	0x0f01010f	
XMSSMT_SHAKE256_W16_H60_D6	Section 5.3	0x01020201	

```
|                             |             |                    |
|  XMSSMT_SHAKE256_W16_H60_D12 | Section 5.3 |     0x02020202     |
+-----------------------------+-------------+--------------------+
```

                              Table 6

   An IANA registration of a signature system does not constitute an
   endorsement of that system or its security.

8. Security Considerations

   A signature system is considered secure if it prevents an attacker
   from forging a valid signature. More specifically, consider a
   setting in which an attacker gets a public key and can learn
   signatures on arbitrary messages of his choice. A signature system
   is secure if, even in this setting, the attacker can not produce a
   new message signature pair of his choosing such that the verification
   algorithm accepts.

   Preventing an attacker from mounting an attack means that the attack
   is computationally too expensive to be carried out. There exist
   various estimates when a computation is too expensive to be done.
   For that reason, this note only describes how expensive it is for an
   attacker to generate a forgery. Parameters are accompanied by a bit
   security value. The meaning of bit security is as follows. A
   parameter set grants b bits of security if the best attack takes at
   least $2^{(b - 1)}$ bit operations to achieve a success probability of
   1/2. Hence, to mount a successful attack, an attacker needs to
   perform 2^b bit operations on average. The given values for bit
   security were estimated according to [HRS16].

8.1. Security Proofs

   A full security proof for the scheme described here can be found in
   [HRS16]. This proof shows that an attacker has to break at least one
   out of certain security properties of the used hash functions and
   PRFs to forge a signature. The proof in [HRS16] considers a
   different initial message compression than the randomized hashing
   used here. We comment on this below. For the original schemes,
   these proofs show that an attacker has to break certain minimal
   security properties. In particular, it is not sufficient to break
   the collision resistance of the hash functions to generate a forgery.

   More specifically, the requirements on the used functions are that F
   and H are post-quantum multi-function multi-target second-preimage
   resistant keyed functions, F fulfills an additional statistical
   requirement that roughly says that most images have at least two
   preimages, PRF is a post-quantum pseudorandom function, H_msg is a

post-quantum multi-target extended target collision resistant keyed
hash function. For detailed definitions of these properties see
[HRS16]. To give some intuition: Multi-function multi-target second
preimage resistance is an extension of second preimage resistance to
keyed hash functions, covering the case where an adversary succeeds
if it finds a second preimage for one out of many values. The same
holds for multi-target extended target collision resistance which
just lacks the multi-function identifier as target collision
resistance already considers keyed hash functions. The proof in
[HRS16] splits PRF into two functions. When PRF is used for
pseudorandom key generation or generation of randomness for
randomized message hashing it is still considered a pseudorandom
function. Whenever PRF is used to generate bitmasks and hash
function keys it is modeled as a random oracle. This is due to
technical reasons in the proof and an implementation using a
pseudorandom function is secure.

The proof in [HRS16] considers classical randomized hashing for the
initial message compression, i.e., $H(r, M)$ instead of $H(r \mathbin{||}$
getRoot(PK) $\mathbin{||}$ index, $M)$. While the classical randomized hashing
used in [HRS16] allows to prove that it is not enough for an
adversary to break the collision resistance of the underlying hash
function, it turns out that an attacker could launch a multi-target
attack even against multiple users at the same time. The reason is
that the adversary attacking u users at the same time learns $u*2^h$
randomized hashes $H(r\_i\_j \mathbin{||} M\_i\_j)$ with signature index i in [0, 2^h
- 1] and user index j in [0, u]. It suffices to find a single pair
$(r*, M*)$ such that $H(r* \mathbin{||} M*) = H(r\_i\_u \mathbin{||} M\_i\_u)$ for one out of the
$u*2^h$ learned hashes. Hence, an attacker can do a brute force search
in time $2^n / u*2^h$ instead of 2^n.

The indexed randomized hashing $H(r \mathbin{||}$ getRoot(PK) $\mathbin{||}$ toByte(idx, n),
M) used in this work makes the hash function calls position- and
user-dependent. This thwarts the above attack because each hash
function evaluation during an attack can only target one of the
learned randomized hash values. More specifically, an attacker now
has to decide which index idx and which root value to use for each
query. This can also be shown formally in the random oracle model.

The given bit security values were estimated based on the complexity
of the best known generic attacks against the required security
properties of the used hash and pseudorandom functions assuming
conventional and quantum adversaries.

8.2. Minimal Security Assumptions

   The security assumptions made to argue for the security of the
   described schemes are minimal. Any signature algorithm that allows
   arbitrary size messages relies on the security of a cryptographic
   hash function. For the schemes described here this is already
   sufficient to be secure. In contrast, common signature schemes like
   RSA, DSA, and ECDSA additionally rely on the conjectured hardness of
   certain mathematical problems.

8.3. Post-Quantum Security

   A post-quantum cryptosystem is a system that is secure against
   attackers with access to a reasonably sized quantum computer. At the
   time of writing this note, whether or not it is feasible to build
   such machine is an open conjecture. However, significant progress
   was made over the last few years in this regard. Hence, we consider
   it a matter of risk assessment to prepare for this case.

   In contrast to RSA, DSA, and ECDSA, the described signature systems
   are post-quantum-secure if they are used with an appropriate
   cryptographic hash function. In particular, for post-quantum
   security, the size of n must be twice the size required for classical
   security. This is in order to protect against quantum square root
   attacks due to Grover's algorithm. It has been shown in [HRS16] that
   variants of Grover's algorithm are the optimal generic attacks
   against the security properties of hash functions required for the
   described scheme.

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Appendix A. WOTS+ XDR Formats

   The WOTS+ signature and public key formats are formally defined using
   XDR [RFC4506] in order to provide an unambiguous, machine readable
   definition. Though XDR is used, these formats are simple and easy to
   parse without any special tools. To avoid the need to convert to and
   from network / host byte order, the enumeration values are all
   palindromes. Note that this representation includes all optional
   parameter sets. The same applies for the XMSS and XMSS^MT formats
   below.

   WOTS+ parameter sets are defined using XDR syntax as follows:

```
    /* ots_algorithm_type identifies a particular
       signature algorithm */

    enum ots_algorithm_type {
      wotsp_reserved     = 0x00000000,
      wotsp_sha2-256_w16 = 0x01000001,
      wotsp_sha2-512_w16 = 0x02000002,
      wotsp_shake128_w16 = 0x03000003,
      wotsp_shake256_w16 = 0x04000004,
    };
```

WOTS+ signatures are defined using XDR syntax as follows:

```
/* Byte strings */

typedef opaque bytestring32[32];
typedef opaque bytestring64[64];

union ots_signature switch (ots_algorithm_type type) {
  case wotsp_sha2-256_w16:
  case wotsp_shake128_w16:
    bytestring32 ots_sig_n32_len67[67];

  case wotsp_sha2-512_w16:
  case wotsp_shake256_w16:
    bytestring64 ots_sig_n64_len18[131];

  default:
    void;   /* error condition */
};
```

WOTS+ public keys are defined using XDR syntax as follows:

```
union ots_pubkey switch (ots_algorithm_type type) {
  case wotsp_sha2-256_w16:
  case wotsp_shake128_w16:
    bytestring32 ots_pubk_n32_len67[67];

  case wotsp_sha2-512_w16:
  case wotsp_shake256_w16:
    bytestring64 ots_pubk_n64_len18[131];

  default:
    void;   /* error condition */
};
```

Appendix B. XMSS XDR Formats

XMSS parameter sets are defined using XDR syntax as follows:

```
/* Byte strings */

typedef opaque bytestring4[4];

/* Definition of parameter sets */

enum xmss_algorithm_type {
  xmss_reserved          = 0x00000000,

   /* 256 bit classical security, 128 bit post-quantum security */

   xmss_sha2-256_w16_h10 = 0x01000001,
   xmss_sha2-256_w16_h16 = 0x02000002,
   xmss_sha2-256_w16_h20 = 0x03000003,

   /* 512 bit classical security, 256 bit post-quantum security */

   xmss_sha2-512_w16_h10 = 0x04000004,
   xmss_sha2-512_w16_h16 = 0x05000005,
   xmss_sha2-512_w16_h20 = 0x06000006,

   /* 256 bit classical security, 128 bit post-quantum security */

   xmss_shake128_w16_h10 = 0x07000007,
   xmss_shake128_w16_h16 = 0x08000008,
   xmss_shake128_w16_h20 = 0x09000009,

   /* 512 bit classical security, 256 bit post-quantum security */

   xmss_shake256_w16_h10 = 0x0a00000a,
   xmss_shake256_w16_h16 = 0x0b00000b,
   xmss_shake256_w16_h20 = 0x0c00000c,
};
```

XMSS signatures are defined using XDR syntax as follows:

```
/* Authentication path types */

union xmss_path switch (xmss_algorithm_type type) {
  case xmss_sha2-256_w16_h10:
  case xmss_shake128_w16_h10:
    bytestring32 path_n32_t10[10];
```

```
      case xmss_sha2-256_w16_h16:
      case xmss_shake128_w16_h16:
        bytestring32 path_n32_t16[16];

      case xmss_sha2-256_w16_h20:
      case xmss_shake128_w16_h20:
        bytestring32 path_n32_t20[20];

      case xmss_sha2-512_w16_h10:
      case xmss_shake256_w16_h10:
        bytestring64 path_n64_t10[10];

      case xmss_sha2-512_w16_h16:
      case xmss_shake256_w16_h16:
        bytestring64 path_n64_t16[16];

      case xmss_sha2-512_w16_h20:
      case xmss_shake256_w16_h20:
        bytestring64 path_n64_t20[20];

      default:
        void;      /* error condition */
      };

      /* Types for XMSS random strings */

      union random_string_xmss switch (xmss_algorithm_type type) {
        case xmss_sha2-256_w16_h10:
        case xmss_sha2-256_w16_h16:
        case xmss_sha2-256_w16_h20:
        case xmss_shake128_w16_h10:
        case xmss_shake128_w16_h16:
        case xmss_shake128_w16_h20:
          bytestring32 rand_n32;

        case xmss_sha2-512_w16_h10:
        case xmss_sha2-512_w16_h16:
        case xmss_sha2-512_w16_h20:
        case xmss_shake256_w16_h10:
        case xmss_shake256_w16_h16:
        case xmss_shake256_w16_h20:
          bytestring64 rand_n64;

        default:
          void;      /* error condition */
      };

      /* Corresponding WOTS+ type for given XMSS type */
```

```
      union xmss_ots_signature switch (xmss_algorithm_type type) {
        case xmss_sha2-256_w16_h10:
        case xmss_sha2-256_w16_h16:
        case xmss_sha2-256_w16_h20:
          wotsp_sha2-256_w16;

        case xmss_sha2-512_w16_h10:
        case xmss_sha2-512_w16_h16:
        case xmss_sha2-512_w16_h20:
          wotsp_sha2-512_w16;

        case xmss_shake128_w16_h10:
        case xmss_shake128_w16_h16:
        case xmss_shake128_w16_h20:
          wotsp_shake128_w16;

        case xmss_shake256_w16_h10:
        case xmss_shake256_w16_h16:
        case xmss_shake256_w16_h20:
          wotsp_shake256_w16;

        default:
          void;        /* error condition */
      };

      /* XMSS signature structure */

      struct xmss_signature {
        /* WOTS+ key pair index */
        bytestring4 idx_sig;
        /* Random string for randomized hashing */
        random_string_xmss rand_string;
        /* WOTS+ signature */
        xmss_ots_signature sig_ots;
        /* authentication path */
        xmss_path nodes;
      };
```

   XMSS public keys are defined using XDR syntax as follows:

```
      /* Types for bitmask seed */

      union seed switch (xmss_algorithm_type type) {
        case xmss_sha2-256_w16_h10:
        case xmss_sha2-256_w16_h16:
        case xmss_sha2-256_w16_h20:
```

```
      case xmss_shake128_w16_h10:
      case xmss_shake128_w16_h16:
      case xmss_shake128_w16_h20:
        bytestring32 seed_n32;

      case xmss_sha2-512_w16_h10:
      case xmss_sha2-512_w16_h16:
      case xmss_sha2-512_w16_h20:
      case xmss_shake256_w16_h10:
      case xmss_shake256_w16_h16:
      case xmss_shake256_w16_h20:
        bytestring64 seed_n64;

      default:
        void;       /* error condition */
    };

    /* Types for XMSS root node */

    union xmss_root switch (xmss_algorithm_type type) {
      case xmss_sha2-256_w16_h10:
      case xmss_sha2-256_w16_h16:
      case xmss_sha2-256_w16_h20:
      case xmss_shake128_w16_h10:
      case xmss_shake128_w16_h16:
      case xmss_shake128_w16_h20:
        bytestring32 root_n32;

      case xmss_sha2-512_w16_h10:
      case xmss_sha2-512_w16_h16:
      case xmss_sha2-512_w16_h20:
      case xmss_shake256_w16_h10:
      case xmss_shake256_w16_h16:
      case xmss_shake256_w16_h20:
        bytestring64 root_n64;

      default:
        void;       /* error condition */
    };

    /* XMSS public key structure */

    struct xmss_public_key {
      xmss_root root;  /* Root node */
      seed SEED;  /* Seed for bitmasks */
    };
```

Appendix C. XMSS^MT XDR Formats

   XMSS^MT parameter sets are defined using XDR syntax as follows:

      /* Byte strings */

      typedef opaque bytestring3[3];
      typedef opaque bytestring5[5];
      typedef opaque bytestring8[8];

      /* Definition of parameter sets */

      enum xmssmt_algorithm_type {
        xmssmt_reserved   = 0x00000000,

        /* 256 bit classical security, 128 bit post-quantum security */

        xmssmt_sha2-256_w16_h20_d2 = 0x01000001,
        xmssmt_sha2-256_w16_h20_d4 = 0x02000002,
        xmssmt_sha2-256_w16_h40_d2 = 0x03000003,
        xmssmt_sha2-256_w16_h40_d4 = 0x04000004,
        xmssmt_sha2-256_w16_h40_d8 = 0x05000005,
        xmssmt_sha2-256_w16_h60_d3 = 0x06000006,
        xmssmt_sha2-256_w16_h60_d6 = 0x07000007,
        xmssmt_sha2-256_w16_h60_d12 = 0x08000008,

        /* 512 bit classical security, 256 bit post-quantum security */

        xmssmt_sha2-512_w16_h20_d2 = 0x09000009,
        xmssmt_sha2-512_w16_h20_d4 = 0x0a00000a,
        xmssmt_sha2-512_w16_h40_d2 = 0x0b00000b,
        xmssmt_sha2-512_w16_h40_d4 = 0x0c00000c,
        xmssmt_sha2-512_w16_h40_d8 = 0x0d00000d,
        xmssmt_sha2-512_w16_h60_d3 = 0x0e00000e,
        xmssmt_sha2-512_w16_h60_d6 = 0x0f00000f,
        xmssmt_sha2-512_w16_h60_d12 = 0x01010101,

        /* 256 bit classical security, 128 bit post-quantum security */

        xmssmt_shake128_w16_h20_d2 = 0x02010102,
        xmssmt_shake128_w16_h20_d4 = 0x03010103,
        xmssmt_shake128_w16_h40_d2 = 0x04010104,
        xmssmt_shake128_w16_h40_d4 = 0x05010105,
        xmssmt_shake128_w16_h40_d8 = 0x06010106,
        xmssmt_shake128_w16_h60_d3 = 0x07010107,
        xmssmt_shake128_w16_h60_d6 = 0x08010108,
        xmssmt_shake128_w16_h60_d12 = 0x09010109,

```
     /* 512 bit classical security, 256 bit post-quantum security */

     xmssmt_shake256_w16_h20_d2  = 0x0a01010a,
     xmssmt_shake256_w16_h20_d4  = 0x0b01010b,
     xmssmt_shake256_w16_h40_d2  = 0x0c01010c,
     xmssmt_shake256_w16_h40_d4  = 0x0d01010d,
     xmssmt_shake256_w16_h40_d8  = 0x0e01010e,
     xmssmt_shake256_w16_h60_d3  = 0x0f01010f,
     xmssmt_shake256_w16_h60_d6  = 0x01020201,
     xmssmt_shake256_w16_h60_d12 = 0x02020202,
   };
```

XMSS^MT signatures are defined using XDR syntax as follows:

```
   /* Type for XMSS^MT key pair index */
   /* Depends solely on h */

   union idx_sig_xmssmt switch (xmss_algorithm_type type) {
     case xmssmt_sha2-256_w16_h20_d2:
     case xmssmt_sha2-256_w16_h20_d4:
     case xmssmt_sha2-512_w16_h20_d2:
     case xmssmt_sha2-512_w16_h20_d4:
     case xmssmt_shake128_w16_h20_d2:
     case xmssmt_shake128_w16_h20_d4:
     case xmssmt_shake256_w16_h20_d2:
     case xmssmt_shake256_w16_h20_d4:
       bytestring3 idx3;

     case xmssmt_sha2-256_w16_h40_d2:
     case xmssmt_sha2-256_w16_h40_d4:
     case xmssmt_sha2-256_w16_h40_d8:
     case xmssmt_sha2-512_w16_h40_d2:
     case xmssmt_sha2-512_w16_h40_d4:
     case xmssmt_sha2-512_w16_h40_d8:
     case xmssmt_shake128_w16_h40_d2:
     case xmssmt_shake128_w16_h40_d4:
     case xmssmt_shake128_w16_h40_d8:
     case xmssmt_shake256_w16_h40_d2:
     case xmssmt_shake256_w16_h40_d4:
     case xmssmt_shake256_w16_h40_d8:
       bytestring5 idx5;

     case xmssmt_sha2-256_w16_h60_d3:
     case xmssmt_sha2-256_w16_h60_d6:
     case xmssmt_sha2-256_w16_h60_d12:
     case xmssmt_sha2-512_w16_h60_d3:
```

```
      case xmssmt_sha2-512_w16_h60_d6:
      case xmssmt_sha2-512_w16_h60_d12:
      case xmssmt_shake128_w16_h60_d3:
      case xmssmt_shake128_w16_h60_d6:
      case xmssmt_shake128_w16_h60_d12:
      case xmssmt_shake256_w16_h60_d3:
      case xmssmt_shake256_w16_h60_d6:
      case xmssmt_shake256_w16_h60_d12:
        bytestring8 idx8;

      default:
        void;       /* error condition */
      };

    union random_string_xmssmt switch (xmssmt_algorithm_type type) {
      case xmssmt_sha2-256_w16_h20_d2:
      case xmssmt_sha2-256_w16_h20_d4:
      case xmssmt_sha2-256_w16_h40_d2:
      case xmssmt_sha2-256_w16_h40_d4:
      case xmssmt_sha2-256_w16_h40_d8:
      case xmssmt_sha2-256_w16_h60_d3:
      case xmssmt_sha2-256_w16_h60_d6:
      case xmssmt_sha2-256_w16_h60_d12:
      case xmssmt_shake128_w16_h20_d2:
      case xmssmt_shake128_w16_h20_d4:
      case xmssmt_shake128_w16_h40_d2:
      case xmssmt_shake128_w16_h40_d4:
      case xmssmt_shake128_w16_h40_d8:
      case xmssmt_shake128_w16_h60_d3:
      case xmssmt_shake128_w16_h60_d6:
      case xmssmt_shake128_w16_h60_d12:
        bytestring32 rand_n32;

      case xmssmt_sha2-512_w16_h20_d2:
      case xmssmt_sha2-512_w16_h20_d4:
      case xmssmt_sha2-512_w16_h40_d2:
      case xmssmt_sha2-512_w16_h40_d4:
      case xmssmt_sha2-512_w16_h40_d8:
      case xmssmt_sha2-512_w16_h60_d3:
      case xmssmt_sha2-512_w16_h60_d6:
      case xmssmt_sha2-512_w16_h60_d12:
      case xmssmt_shake256_w16_h20_d2:
      case xmssmt_shake256_w16_h20_d4:
      case xmssmt_shake256_w16_h40_d2:
      case xmssmt_shake256_w16_h40_d4:
      case xmssmt_shake256_w16_h40_d8:
      case xmssmt_shake256_w16_h60_d3:
      case xmssmt_shake256_w16_h60_d6:
```

```
      case xmssmt_shake256_w16_h60_d12:
        bytestring64 rand_n64;

      default:
        void;       /* error condition */
    };

    /* Type for reduced XMSS signatures */

    union xmss_reduced (xmss_algorithm_type type) {
      case xmssmt_sha2-256_w16_h20_d2:
      case xmssmt_sha2-256_w16_h40_d4:
      case xmssmt_sha2-256_w16_h60_d6:
      case xmssmt_shake128_w16_h20_d2:
      case xmssmt_shake128_w16_h40_d4:
      case xmssmt_shake128_w16_h60_d6:
        bytestring32 xmss_reduced_n32_t77[77];

      case xmssmt_sha2-256_w16_h20_d4:
      case xmssmt_sha2-256_w16_h40_d8:
      case xmssmt_sha2-256_w16_h60_d12:
      case xmssmt_shake128_w16_h20_d4:
      case xmssmt_shake128_w16_h40_d8:
      case xmssmt_shake128_w16_h60_d12:
        bytestring32 xmss_reduced_n32_t72[72];

      case xmssmt_sha2-256_w16_h40_d2:
      case xmssmt_sha2-256_w16_h60_d3:
      case xmssmt_shake128_w16_h40_d2:
      case xmssmt_shake128_w16_h60_d3:
        bytestring32 xmss_reduced_n32_t87[87];

      case xmssmt_sha2-512_w16_h20_d2:
      case xmssmt_sha2-512_w16_h40_d4:
      case xmssmt_sha2-512_w16_h60_d6:
      case xmssmt_shake256_w16_h20_d2:
      case xmssmt_shake256_w16_h40_d4:
      case xmssmt_shake256_w16_h60_d6:
        bytestring64 xmss_reduced_n32_t141[141];

      case xmssmt_sha2-512_w16_h20_d4:
      case xmssmt_sha2-512_w16_h40_d8:
      case xmssmt_sha2-512_w16_h60_d12:
      case xmssmt_shake256_w16_h20_d4:
      case xmssmt_shake256_w16_h40_d8:
      case xmssmt_shake256_w16_h60_d12:
        bytestring64 xmss_reduced_n32_t136[136];
```

```
        case xmssmt_sha2-512_w16_h40_d2:
        case xmssmt_sha2-512_w16_h60_d3:
        case xmssmt_shake256_w16_h40_d2:
        case xmssmt_shake256_w16_h60_d3:
           bytestring64 xmss_reduced_n32_t151[151];

        default:
           void;       /* error condition */
      };

      /* xmss_reduced_array depends on d */

      union xmss_reduced_array (xmss_algorithm_type type) {
        case xmssmt_sha2-256_w16_h20_d2:
        case xmssmt_sha2-512_w16_h20_d2:
        case xmssmt_sha2-256_w16_h40_d2:
        case xmssmt_sha2-512_w16_h40_d2:
        case xmssmt_shake128_w16_h20_d2:
        case xmssmt_shake256_w16_h20_d2:
        case xmssmt_shake128_w16_h40_d2:
        case xmssmt_shake256_w16_h40_d2:
           xmss_reduced xmss_red_arr_d2[2];

        case xmssmt_sha2-256_w16_h60_d3:
        case xmssmt_sha2-512_w16_h60_d3:
        case xmssmt_shake128_w16_h60_d3:
        case xmssmt_shake256_w16_h60_d3:
           xmss_reduced xmss_red_arr_d3[3];

        case xmssmt_sha2-256_w16_h20_d4:
        case xmssmt_sha2-512_w16_h20_d4:
        case xmssmt_sha2-256_w16_h40_d4:
        case xmssmt_sha2-512_w16_h40_d4:
        case xmssmt_shake128_w16_h20_d4:
        case xmssmt_shake256_w16_h20_d4:
        case xmssmt_shake128_w16_h40_d4:
        case xmssmt_shake256_w16_h40_d4:
           xmss_reduced xmss_red_arr_d4[4];

        case xmssmt_sha2-256_w16_h60_d6:
        case xmssmt_sha2-512_w16_h60_d6:
        case xmssmt_shake128_w16_h60_d6:
        case xmssmt_shake256_w16_h60_d6:
           xmss_reduced xmss_red_arr_d6[6];

        case xmssmt_sha2-256_w16_h40_d8:
        case xmssmt_sha2-512_w16_h40_d8:
        case xmssmt_shake128_w16_h40_d8:
```

```
      case xmssmt_shake256_w16_h40_d8:
        xmss_reduced xmss_red_arr_d8[8];

      case xmssmt_sha2-256_w16_h60_d12:
      case xmssmt_sha2-512_w16_h60_d12:
      case xmssmt_shake128_w16_h60_d12:
      case xmssmt_shake256_w16_h60_d12:
        xmss_reduced xmss_red_arr_d12[12];

      default:
        void;       /* error condition */
    };

    /* XMSS^MT signature structure */

    struct xmssmt_signature {
      /* WOTS+ key pair index */
      idx_sig_xmssmt idx_sig;
      /* Random string for randomized hashing */
      random_string_xmssmt randomness;
      /* Array of d reduced XMSS signatures */
      xmss_reduced_array;
    };
```

   XMSS^MT public keys are defined using XDR syntax as follows:

```
    /* Types for bitmask seed */

    union seed switch (xmssmt_algorithm_type type) {
      case xmssmt_sha2-256_w16_h20_d2:
      case xmssmt_sha2-256_w16_h40_d4:
      case xmssmt_sha2-256_w16_h60_d6:
      case xmssmt_sha2-256_w16_h20_d4:
      case xmssmt_sha2-256_w16_h40_d8:
      case xmssmt_sha2-256_w16_h60_d12:
      case xmssmt_sha2-256_w16_h40_d2:
      case xmssmt_sha2-256_w16_h60_d3:
      case xmssmt_shake128_w16_h20_d2:
      case xmssmt_shake128_w16_h40_d4:
      case xmssmt_shake128_w16_h60_d6:
      case xmssmt_shake128_w16_h20_d4:
      case xmssmt_shake128_w16_h40_d8:
      case xmssmt_shake128_w16_h60_d12:
      case xmssmt_shake128_w16_h40_d2:
      case xmssmt_shake128_w16_h60_d3:
        bytestring32 seed_n32;
```

```
      case xmssmt_sha2-512_w16_h20_d2:
      case xmssmt_sha2-512_w16_h40_d4:
      case xmssmt_sha2-512_w16_h60_d6:
      case xmssmt_sha2-512_w16_h20_d4:
      case xmssmt_sha2-512_w16_h40_d8:
      case xmssmt_sha2-512_w16_h60_d12:
      case xmssmt_sha2-512_w16_h40_d2:
      case xmssmt_sha2-512_w16_h60_d3:
      case xmssmt_shake256_w16_h20_d2:
      case xmssmt_shake256_w16_h40_d4:
      case xmssmt_shake256_w16_h60_d6:
      case xmssmt_shake256_w16_h20_d4:
      case xmssmt_shake256_w16_h40_d8:
      case xmssmt_shake256_w16_h60_d12:
      case xmssmt_shake256_w16_h40_d2:
      case xmssmt_shake256_w16_h60_d3:
         bytestring64 seed_n64;

      default:
         void;        /* error condition */
   };


   /* Types for XMSS^MT root node */

   union xmssmt_root switch (xmssmt_algorithm_type type) {
      case xmssmt_sha2-256_w16_h20_d2:
      case xmssmt_sha2-256_w16_h20_d4:
      case xmssmt_sha2-256_w16_h40_d2:
      case xmssmt_sha2-256_w16_h40_d4:
      case xmssmt_sha2-256_w16_h40_d8:
      case xmssmt_sha2-256_w16_h60_d3:
      case xmssmt_sha2-256_w16_h60_d6:
      case xmssmt_sha2-256_w16_h60_d12:
      case xmssmt_shake128_w16_h20_d2:
      case xmssmt_shake128_w16_h20_d4:
      case xmssmt_shake128_w16_h40_d2:
      case xmssmt_shake128_w16_h40_d4:
      case xmssmt_shake128_w16_h40_d8:
      case xmssmt_shake128_w16_h60_d3:
      case xmssmt_shake128_w16_h60_d6:
      case xmssmt_shake128_w16_h60_d12:
         bytestring32 root_n32;

      case xmssmt_sha2-512_w16_h20_d2:
      case xmssmt_sha2-512_w16_h20_d4:
      case xmssmt_sha2-512_w16_h40_d2:
      case xmssmt_sha2-512_w16_h40_d4:
      case xmssmt_sha2-512_w16_h40_d8:
```

```
    case xmssmt_sha2-512_w16_h60_d3:
    case xmssmt_sha2-512_w16_h60_d6:
    case xmssmt_sha2-512_w16_h60_d12:
    case xmssmt_shake256_w16_h20_d2:
    case xmssmt_shake256_w16_h20_d4:
    case xmssmt_shake256_w16_h40_d2:
    case xmssmt_shake256_w16_h40_d4:
    case xmssmt_shake256_w16_h40_d8:
    case xmssmt_shake256_w16_h60_d3:
    case xmssmt_shake256_w16_h60_d6:
    case xmssmt_shake256_w16_h60_d12:
      bytestring64 root_n64;

    default:
      void;      /* error condition */
  };

  /* XMSS^MT public key structure */

  struct xmssmt_public_key {
    xmssmt_root root;  /* Root node */
    seed SEED;  /* Seed for bitmasks */
  };
```

Appendix D. Changed since draft-irtf-cfrg-xmss-hash-based-signatures-03

   1: Pseudocode examples now include input and output explicitly.

   2: Changed the addresses for the hash function address scheme.

   2.1: Addresses are now 32 bytes long.

   2.2: Some address elements were increased in size, especially tree
   address, which is now 64 bits long.

   3: R = PRF(SK, idx) instead of R = PRF(SK, M).

   4: Changes for hash functions:

   4.1: ChaCha20 is no longer used.

   4.2: SHA2-256 parameter sets are now mandatory, while SHA2-512 sets
   are optional.

   4.3: Added optional SHA-3 support.

5: Former message digest length m was removed. Just as with the proposed parameter sets it is set to be of the same length as the security parameter n throughout the whole document.

6: PRF_m (now called "PRF" using n) now accepts an n-byte string instead of a string with arbitrary length (please see point 5).

7: Where applicable (formerly algorithms 11, 12 and 15), hashing functions where adapted as follows (please also note change 8.3 below): H_m((toByte(idx_sig, m) || r), M) replaced by H_msg(r || getRoot(PK) || (toByte(idx_sig, n)), M) or H_msg(r || getRoot(SK) || (toByte(idx_sig, n)), M), accordingly. Replaced H_m((toByte(idx_sig, m) || getR(Sig_MT)), M) by H_msg(getR(Sig_MT) || getRoot(PK_MT) || (toByte(idx_sig, n)), M), likewise. Please note that the naming for the hash function was adapted due to the new input and m = n.

8: Adapted several algorithms:

8.1: To avoid confusion between len_2 and len_2_bytes output, base_w was changed to always return arrays of a given number of elements.

8.2: Instead of algorithms to only generate public keys for XMSS and XMSS^MT, we now show key generation algorithms XMSS_keyGen and XMSSMT_keyGen (Algorithms 10 and 15) which outline basic secret key generation as well.

8.3: The functions omitting hashing for XMSS^MT (marked by "wo_hash") were removed. Instead the corresponding functions were adapted. Now the new treeSig (algorithm 11) and the adapted XMSS_rootFromSig (algorithm 13) suffice for their needed use. Signature generation and verification algorithms were adapted accordingly.

9: Extension of the security section.

10: Several textual fixes and extensions.

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